MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The order and degree of the partial differential equation $\frac{\partial^2 z}{\partial x\,\partial y} + \left(\frac{\partial z}{\partial x}\right)^2 = 0$ is

(a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 2, 2.

(ii) The solution of $zp = -x$ (where $p = \frac{\partial z}{\partial x}$) is

(a) $\varphi(y, x^2 + z^2) = 0$ (b) $\varphi\left(x, \frac{z}{y}\right) = 0$

(c) $\varphi\left(z, \frac{x^3}{y}\right) = 0$ (d) $\varphi(yx + z, x^2 + z^2) = 0$.

(iii) For $f(z) = \frac{\sin z}{z}$, the point $z = 0$ is

(a) an essential singularity (b) a non-isolated singularity

(c) a removable singularity (d) a pole of order 2.

(iv) The period of the function $f(x) = \cos x + \cos\sqrt{8}\,x$ is

(a) $\pi + \frac{\pi}{2\sqrt{2}}$. (b) $2(1 + \sqrt{2})\pi$

(c) $2\sqrt{2}\pi$ (d) 2π.

(v) The function $f(z) = u(x, y) + iv(x, y)$ is an analytic function of the complex variable $z = x + iy$ where $i = \sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ may be expressed as

(a) $-x^2 + y^2 + constant$ (b) $x^2 - y^2 + constant$
(c) $x^2 + y^2 + constant$ (d) $-(x^2 + y^2) + constant$.

(vi) For the differential equation $x^2(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(a) $x = 1$ is an ordinary point (b) $x = 1$ is a singular point
(c) $x = 0$ is an ordinary point (d) $x = 0$ is not a singular point.

(vii) If $F[f(x)] = F(s)$, then $F[\cos ax\, f(x)] = \frac{1}{2}[F(s+a) + F(s-a)]$. This property of the Fourier transform is known as

(a) linearity property (b) shifting property
(c) modulation property (d) scaling property.

(viii) The value of $\int_{-1}^{1} P_2(x)P_5(x)\,dx$ (where $P_2(x)$ and $P_5(x)$ are Legendre polynomials) is

(a) 3 (b) 2 (c) 1 (d) 0.

(ix) The complete integral of $(p-q)(z - px - qy) = 1$ (where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$) is

(a) $z = ax^2 + by^2 + \frac{1}{a-b}$ (b) $z = ax + by + \frac{1}{a-b}$
(c) $z = axy + \frac{1}{a-b}$ (d) $z = (a-x) + (b-y) + \frac{1}{a-b}$.

(x) The function $f(z) = \bar{z}$ is

(a) continuous at $z = 0$ (b) differentiable at $z = 0$
(c) analytic (d) continuous, except at $z = 0$.

Group – B

2. (a) Show that the function $u(x, y) = x^3 - 3xy^2$ is harmonic. Construct the corresponding analytic function $f(z) = u(x, y) - i\,v(x, y)$.

(b) Find $\oint_C \frac{\tan\frac{z}{2}}{(z-a)^2}\,dz$ where C is the boundary of the square whose sides lie along $x = \pm 2, y = \pm 2$ described in the positive sense and $-2 < a < 2$.

6 + 6 = 12

3. (a) Find the Laurent's series expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$ in all possible regions.

(b) Evaluate $\oint_c \frac{e^{2z}}{(z-1)(z-2)} dz$ where C is the circular path $|z| = 3$.

7 + 5 = 12

Group – C

4. (a) Write the half range sine series of $f(x) = 2 - x$ for $x \in [0, 2]$, and hence find the sum of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.

(b) Find $f(x)$ if its Fourier cosine transform is $F_c(s) = \begin{cases} a - \frac{s}{2}, for\ s < 2a \\ 0\ \ , for\ s \geq 2a \end{cases}$.

6 + 6 = 12

5. (a) Using the cosine series expansion of the function $f(x) = x$ in $(0, \pi)$, find the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} \ldots.$

(b) Find the Fourier transform of $f(x) = \begin{cases} x, |x| \leq 3 \\ 0, |x| > 3 \end{cases}$. Also, find the Fourier transform of $g(x) = 2f(3x)$.

6 + 6 = 12

Group – D

6. (a) Find the power series of $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ about $x = 0$.

(b) Prove that $P_n(1) = 1$, where $P_n(x)$ is a Legendre polynomial.

8 + 4 = 12

7. (a) State the generating function of the Bessel's function $J_n(x)$. Use this to prove that $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$.

(b) Express $x^3 + x^2$ in terms of Legendre polynomials $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$.

6 + 6 = 12

Group – E

8. (a) Eliminate the arbitrary function f from $z = f(x^2 - y^2)$ to obtain a partial differential equation.

(b) Find the complete integral of $q = (z + px)^2$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

5 + 7 = 12

9. (a) Find the general solution of the following partial differential equation: $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$, where $D \equiv \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$.

(b) Solve $\frac{\partial u}{\partial x} = 8\frac{\partial u}{\partial y}$ if $u(0, y) = 7e^{-4y}$ by the method of separation of variables.

6 + 6 = 12

Department & Section	Submission Link
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