B.TECH/AEIE /ECE/IT/3RD SEM/MATH 2002 (BACKLOG)/2020

NUMERICAL & STATISTICAL METHODS
(MATH 2002)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The order of convergence of bisection method is
(a) linear (b) quadratic
(c) cubic (d) 1.5.

(ii) $\Delta.\nabla =$ (the notations have their usual meanings)
(a) $\Delta - \nabla$ (b) $\nabla - \Delta$ (c) $\Delta + \nabla$ (d)0.

(iii) Trapezoidal rule for evaluation of $\int_a^b f(x)dx$ requires the interval (a, b) to be divided into:
(a) $2n$ subintervals of equal width
(b) $(2n + 1)$ subintervals of equal width
(c) any number of subintervals of equal width
(d) $(2n + 1)$ subintervals of unequal width.

(iv) A system of equations $AX = b$, where $A = (a_{ij})_{n\times n}$, is said to be diagonally dominant if
(a)$|a_{ii}| \neq \sum_{\substack{j=1 \\ i \ j}}^{n} |a_{ij}|$ for all i (b) $|a_{ii}| < \sum_{\substack{j=1 \\ i \ j}}^{n} |a_{ij}|$ for all $i \neq$ j
(c) $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ for all $i \neq$ j (d) $|a_{ii}| \leq \sum_{j=1}^{n} |a_{ij}|$ for all i .

(v) A random variable X is uniformly distributed in the interval $[a, b]$. Then the mean of X is
(a) $\frac{1}{b-a}$ (b) $\frac{a+b}{2}$ (c) $\frac{b-1}{a-1}$ (d) $\frac{b}{a}$

(vi) If two fair coins are flipped and at least one of the outcomes is known to be head, the probability that both outcomes are heads is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$.

(vii) If a Poisson variate X is such that $P(X = 1) = P(X = 2)$, then $P(X = 0)$ is

(a) e^{-1} (b) e^{-4} (c) e^{-2} (d) 1.

(viii) Consider a random variable X takes the values $+1$ and -1 with probability 0.5 each. The values of the distribution function $F(x)$ at $x = -1$ is

(a) 0 (b) 0.5 (c) 0.75 (d) 1.

(ix) Let X be a random variable. Then the combination of $E(X)$ and $E(X^2)$ is **NOT** possible for the random variable X is

(a) 0 and 1 (b) $\frac{1}{4}$ and $\frac{1}{5}$ (c) 4 and 5 (d) $\frac{1}{2}$ and $\frac{1}{3}$.

(x) The mode and median of the observation 4, 6, 6, 8, 3, 8, 8 and 4 are

(a) 8 and 6 (b) 8.5 and 6.5

(c) 5 and 7 (d) 4 and 3

Group – B

2. (a) Find a positive real root of the equation $x^3 - x - 4 = 0$ by Regula-Falsi method correct upto 3 decimal places.

(b) Solve the given system of equations using Gauss Elimination method

$$x + 2y + 3z = 6$$
$$2x + 4y + z = 7$$
$$3x + 2y + 9z = 14$$

6 + 6 = 12

3. (a) Solve the following system of equations

$$3x_1 + 2x_2 - 4x_3 = 12$$
$$-x_1 + 5x_2 + 2x_3 = 1$$
$$2x_1 - 3x_2 + 4x_3 = -3$$

by LU factorization method.

(b) Find a positive value of $(17)^{\frac{1}{3}}$ correct upto four decimal places by Newton-Raphson method.

7 + 5 = 12

Group – C

4. (a) Find the missing terms in the following table:

x	0	1	2	3	4	5
$f(x)$	0	f_1	8	15	f_4	35

(b) Using Euler's method, find an approximate value of y at 0.5, given that

$$\frac{dy}{dx} = 3x + y^2, \qquad y(0) = 1, \qquad h = 0.1.$$

6 + 6 = 12

5. (a) Evaluate $\int_0^1 \frac{1}{1+x}\, dx$ by using Simpson's $\frac{1}{3}$rd rule taking eleven ordinates and hence find the value of $\log_e 2$ correct upto five significant figures.

(b) Find the value of $\sqrt{2}$ using Newton's forward interpolation formula for the given data:

x	1.9	2.1	2.3	2.5	2.7
$f(x) = \sqrt{x}$	1.3784	1.4491	1.5166	1.5811	1.6432

6 + 6 = 12

Group – D

6. (a) In a certain college 25% of the boys and 10% of the girls are studying Mathematics. The girls constitute 60% of the total number of students. If a student is chosen randomly and found to be studying Mathematics, then determine the probability that the student is a girl.

(b) 100 prizes will be given in a lottery of 10000 tickets. Find the minimum number of tickets a person has to buy in order that the probability of his wining at least one prize is greater than $\frac{1}{2}$.

6 + 6 = 12

7. (a) Suppose that 5 men out of 100 and 25 women out of 10000 are colour-blind. A colour-blind person is chosen at random. What is the probability of his being male?(Assume males and females to be in equal numbers).

(b) Two newspapers X and Y are published in a certain city. It is estimated from a survey that 16% read X, 14% read Y and 5% read both the newspapers. Find the probabilities that a randomly selected person (i) does not read any newspaper,(ii) read only Y.

(c) An event A is independent of itself. What are the possible values of $P(A)$?

5 + 4 + 3 = 12

Group – E

8. (a) Verify whether the function

$$f(x) = \begin{cases} |x|, & -1 < x < 1 \\ 0, & elsewhere \end{cases}$$

is a probability density function. If so, find the corresponding distribution function.

(b) A random variable X follows binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming the non-zero value of the variable.

(c) If X is uniformly distributed over $(1, 2)$, find U so that

$$P(X > U + \bar{X}) = \frac{1}{6}, \text{ where } \bar{X} = E(X).$$

6 + 3 + 3 = 12

9. (a) Assuming that the height distribution of a group is normal, find the mean and standard deviation if 84% of the men have heights less than 65.2 inches and 68% have heights lying between 62.8 and 65.2 inches.

[Given $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.9} e^{-\frac{t^2}{2}}\,dt = 0.84$ and $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-0.9} e^{-\frac{t^2}{2}}\,dt = 0.16$]

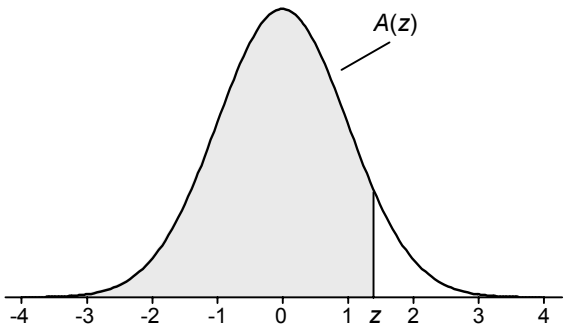
(b) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible:
Regression equations: $8x - 10y + 66 = 0,\ 40x - 18y - 214 = 0.$
(i) Identify which one is the regression line of y on x.
(ii) Find the mean values of x and y.
(iii) Find the correlation coefficient between x and y.

6+6 = 12

Department & Section	Submission Link
AEIE/ECE/IT	https://classroom.google.com/c/MjA0MDA1OTQzNzQz/a/Mjc0NTY1OTM0MDM5/details

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							