2014

PHYSICS–I

(PH 1001)

Time Alloted : 3 Hours *Full Marks : 70*

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternative for the following : **[10×1=10]**

i) The velocity of a particle executing SHM is given by $v = v_0\left[1-\frac{x^2}{B^2}\right]^{\frac{1}{2}}$ where x represents displacement of the particle. The acceleration of the particle will be

(a) $-B^2x$ (b) $-Bv_0x$

(c) $-\frac{v_0}{B^2}x$ (d) $-\frac{v_0^2}{B^2}x$

ii) If λ_L and λ_K are wavelengths of L_α and K_α, x-rays respectively, then

(a) $\lambda_L > \lambda_K$ (b) $\lambda_L = \lambda_K$

(c) $\lambda_L < \lambda_K$ (d) $\lambda_L = 2\lambda_K$

iii) The forced harmonic oscillators have displacement amplitude at frequencies $\omega_1 = 400$ sec^{-1} and $\omega_2 = 600$ sec^{-1}. The resonant frequency at which the displacement amplitude is maximum will be

(a) 480 sec^{-1} (b) 500 sec^{-1}

(c) 510 sec^{-1} (d) 550 sec^{-1}

iv) An x-ray photon collides with a free electron at rest and is scattered at 60° and the recoil angle of the electron is 54.34°. The frequency of the photon is

(a) 0.002×10^{15} MHz (b) 0.02×10^{15} MHz

(c) 0.2×10^{15} MHz (d) 2×10^{15} MHz

v) The minimum thickness of a calcite plate which would convert plane polarized light into circularly polarized light ($\mu_e = 1.486$, $\mu_0 = 1.658$ and $\lambda = 5890$ Å) is

(a) 0.856×10^{-4} cm (b) 1.702×10^{-4} cm

(c) 1.856×10^{-4} cm (d) 2.702×10^{-5} cm

vi) A stationary body explodes into two fragments each of mass 1.0 kg that move apart at speed of 0.6c relative to the original body. The mass of the original body is

(a) 2.5 Kg. (b) 2.0 Kg.

(c) 1.5 Kg. (d) 1.0 Kg.

vii) In Young's double slit experiment the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe-width is

(a) unchanged (b) halved

(c) doubled (d) quadrupled

viii) The intensity of principal maxima in the spectrum of grating with N number of lines is proportional to

(a) $\frac{1}{N}$ (b) N

(c) N^2 (d) $\frac{1}{N^2}$

ix) A particle of mass m Kg and charge Q Coulomb is associated through V Volt, then the de Broglie wavelength associated with it, is

(a) $\lambda = h/\sqrt{mV}$ metre (b) $\lambda = h/\sqrt{2mQ}$ metre

(c) $\lambda = h/\sqrt{2mQV}$ metre (d) $\lambda = h/\sqrt{2mV}$ metre

x) If a and r be respectively the lattice constant and radius of an atom in a simple cubic (s.c.) structure then

(a) $a = \frac{4r}{\sqrt{3}}$ (b) $a = 2r$

(c) $a = \frac{2r}{\sqrt{2}}$ (d) $a = \frac{r}{2}$

GROUP - B

2. (a) How does a diffraction spectra differ from interference spectra?

(b) Deduce an expression for the intensity pattern due to Fraunhofer diffraction by single slit with necessary theory. Also show your result graphically.

(c) A thin mica sheet of r.i. 1.58 is used to cover one slit of a double-slit arrangement. The central fringe is now occupied by what had been the eighth bright fringe before the mica was introduced. What is the thickness of mica sheet if the wavelength of light illuminating the slits is 640 nm?

(d) Describe the state of polarization of the light wave represented by

$$\vec{E}(z,t) = \hat{i}\,E_0 \sin(kz - \omega t) - \hat{j}\,E_0 \sin(kz - \omega t).$$

2+(5+1)+2+2 = 12

3. (a) Suppose you have two crossed polaroids. Now a third Polaroid is inserted between them with its transmission axis at 60° to the transmission axis of each of the two crossed polaroids. Determine the intensity of emergent light in terms of the intensity I_0 of the incidet unpolarised light.

(b) Show that the ratio of stimulated and spontaneous emission in connection with laser transition is equal to

$$\frac{1}{e^{\frac{h\nu}{KT}} - 1}.$$

(c) A beam of linearly polarized light is changed into circularly polarized light by passing it through a slice of crystal 0.005 cm thick. Calculate the difference in the refractive indices of the two rays in the crystal, assuming this to be the minimum thickness to produce the effect and wavelength of light is 6×10^{-5} cm.

(d) A plane transmission grating has 7000 lines/cm. It is used to obtain a spectrum of light from sodium lamp in second order. Calculate the angular separation between two sodium lines 5893 Å and 5896 Å.

(e) In optical fibre, the core material has refractive index 1.6 and clad material has refractive index 1.3. Calculate the value of the critical angle and acceptance angle.

2+4+2+2+2 = 12

GROUP - C

4. (a) Three simple harmonic motions of the same frequency act on a particle simultaneously in the same direction. Their amplitudes are 1.0, 2.0 and 1.5 cm, respectively. The phase angle of the second with respect to the first is 60°, and that of the third with respect to the second is 30°. Obtain the resultant amplitude and phase angle relative to the first.

(b) Show that a uniform circular motion is equivalent to two mutually perpendicular simple harmonic motions of the same frequency but with a phase difference of 90°.

(c) A telephone diaphragm executes damped vibration. For a displacement of 0.01 mm the restoring force is 10^4 dynes. The retarding force is 4×10^3 dynes for a velocity of 2.5 cm/s. If the effective mass of the diaphragm, treated as a particle, is 1 gm, what is the frequency? Write down its equation of motion.

(d) A weekly damped harmonic oscillator is driven by a force $F = F_0 \cos\omega t$. If m is the mass and R is the damping force per unit velocity, show that the width of the amplitude resonance curve is approximately $\sqrt{3}$ R/m, the width is measured between two frequencies where the amplitude equals half the maximum value.

(e) Show that the function $\psi(x,t) = f_1(ct - x) + f_2(ct + x)$ satisfies the one dimensional differential wave equation.

2+2+(2+1)+3+2 = 12

5. (a) A mechanical oscillator of mass 'm' and stiffness constant 'k' is subjected to a damping force proportional to its velocity and an external periodic force $F = F_0 \cos\omega t$. Show that, in the steady state, the time-averaged power supplied to the oscillator by the external force equal the time averaged power dissipated through damping.

(b) A vibrator of mass 1 gm is acted upon by a restoring force of 10^7 dyne/cm, a retarding force of 4×10^3 dyne.sec/cm., and a driving force of $10^7 \cos\omega t$ dyne. Find the maximum possible amplitude.

(c) A particle of mass 1 kg moving along the x-axis is acted on by a force 4x Newton towards the origin, and a damping force 4x Newton. The particle is initially at rest at x = 1m. Find the instant of time, when the velocity of the particle is maximum. What is the maximum velocity?

(d) Distinguish between amplitude resonance and velocity resonance. **4+2+2+2+2 = 12**

Group - D

6. (a) Draw a diagram showing the observed black body spectrum and the Rayleigh-Jeans classical prediction. Write down the expression for Planck's distribution and also state Planck's basic assumptions.

(b) Show that the Wien displacement law is a consequence of the Planck black body radiation law.

(c) If m_0 be the rest mass of an electron and λ_C the Crompton wavelength, then prove that, the maximum recoil energy of a free electron when struck by a photon of wavelength λ is $E = \dfrac{2m_0c^2\lambda_C^2}{\lambda^2 + 2\lambda\lambda_C}$.

(d) Prove that a free electron cannot absorb a photon completely in Crompton scattering.

(1+1+2)+3+3+2 = 12

7. (a) Calculate the de Broglie wavelength of a α-particle accelerated through a potential difference of 20 volts. Given : Mass of proton = 1.67×10^{-24} gm.

(b) Use Heisenberg Uncertainty principle to prove that the electron cannot exist in the nucleus.

(c) The phase velocity of ocean waves is $\sqrt{g\lambda/2\pi}$, where g is the acceleration due to gravity. Find the group velocity of ocean waves.

(d) An X-ray tube is operated at an anode potential of 10 kV and anode current of 15 mA. Calculate (i) number of electrons hitting the anode per second and (ii) the minimum wavelength produced by the X-ray tube.

(e) What is pair production? An electron and a positron are moving side by side in the +x direction at 0.50c (c = velocity of light) when they annihilate each other. Two photons are produced that move along the x-axis. What is the energy of each photon?

2+2+2+2+(1+3) = 12

GROUP - E

8. (a) Show that the maximum radius of the sphere that can just fit into the void at the centre of the unit cell of f.c.c. structure having atoms of radius r is 0.414r.

(b) In a crystal whose primitives are 1.2 Å, 1.8 Å, 2 Å, a plane (2 3 1) cuts on intercept 1.2 Å along X-axis. Find the lengths of intercepts along Y and Z-axis.

(c) Show that the distance between successive planes of Miller indices (h k l) is given by $d_{hkl} = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-\frac{1}{2}}$,

where a, b, c are the primitive vectors of the lattice. In a simple cubic crystal, find the ratio of the spacings (1 1 0) and (1 1 1) planes.

(d) Can Miller indices of a plane in a regular crystal be irrational? Explain. **2+3+(3+2)+(1+1) = 12**

9. (a) The wavelength of the K_α line of Ag is 0.563 Å. The radiation from an Ag target is analysed with a Bragg spectrometer using a calcite crystal (a simple cube of lattice constant 3.02945 Å). Determine the angle of reflection for the first order. What is the highest order for which this line may be observed?

(b) Sketch the following directions and planes in a cubic unit cell

(i) $[1\ \bar{1}\ 1]$ (ii) $[\bar{1}\ 0\ 1]$

(c) What is unit cell? Find the packing factor for body entered cubic (b.c.c.) structure.

(d) Copper has f.c.c. structure and the atomic radius 1.278 Å. Calculate the density of copper crystal. Given atomic wt. of copper = 63.5, Avogadro's number = 6.023×10^{23}.

(2+2)+3+(1+2)+2 = 12