Group – E

8. (a) Classify the following partial differential equations:
(i) $u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = x^2 + y^2$
(ii) $u_{tt} + (5 + 2x^2)u_{xt} + (1 + x^2)(4 + x^2)u_{xx} = 0$.

(b) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ numerically for the following mesh (Fig.1) with uniform spacing and with specified boundary conditions as shown below:

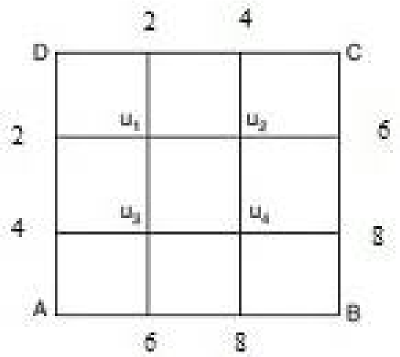

Fig.1

(2 + 2) + 8 = 12

9. (a) State the condition for the linear second order partial differential equation $Au_{xx} + 2Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu + G = 0$ to be elliptic, hyperbolic or parabolic.

(b) Solve the boundary value problem for the Poisson equation $u_{xx} + u_{yy} = x^2 - 1, |x| \leq 1, |y| \leq 1$.
Consider $u = 0$ on the boundary of the square. Use the five point formula with square mesh of width $h = 1/2$.

3 + 9 = 12

COMPUTATIONAL METHODS IN ENGINEERING
(MECH 4142)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The numerical solution of a mathematical model is
(a) exact solution (b) erroneous solution
(c) approximate solution (d) both (b) and (c)

(ii) The number of significant digits in 3.0509 is
(a) 3 (b) 5 (c) 6 (d) 7.

(iii) Nine readings are taken from a measurement. It is seen that they are very close to each other and also closer to the true measurement. They are
(a) precise but not accurate (b) accurate but not precise
(c) accurate as well as precise (d) neither accurate nor precise.

(iv) Which one is the iterative method to solve linear equations
(a) Gauss Elimination (b) Gauss Jordon
(c) Matrix inversion (d) Gauss Seidel.

(v) During the solution of linear equations by Gauss Elimination method if pivot element is seen zero, the solution is
(a) not possible
(b) possible but inaccurate result may come
(c) possible with partial pivoting
(d) possible with infinite many solution.

(vi) In Gauss-Seidel method
(a) the value of one variable is used immediately to the next iteration
(b) the same values of the variables are used throughout one complete iteration
(c) iterations are performed by any arbitrary value of a variable
(d) the values found from one complete iteration are never used further.

(vii) Simpson's 1/3rd rule for performing numerical integration corresponds to the fitting of
(a) first order polynomial (b) second order polynomial
(c) third order polynomial (d) fourth order polynommial.

(viii) Trapezoidal rule for performing numerical integration corresponds to the fitting of
(a) fourth order polynomial (b) second order polynomial
(c) third order polynomial (d) first order polynomial.

(ix) An example of initial boundary value problem governed by linear second order partial differential equation is
(a) $u_{xx} + u_{yy} = 0$ (b) $u_{xx} + u_{yy} = G(x, y)$
(c) $u_t = c^2 u_{xx}$ (d) both (a) and (b).

(x) The first order difference approximation for y_i' based on forward difference is
(a) $y_i' = [y_{i+1} - y_i]/2h$ (b) $y_i' = [y_i - y_{i-1}]/h$
(c) $y_i' = [y_{i+1} - y_{i-1}]/2h$ (d) $y_i' = [y_{i+1} - y_i]/h$

Group – B

2. (a) Suppose that a spherical droplet of liquid evaporates at a rate that is proportional to its surface area.

$$\frac{dV}{dt} = -kA$$

Where V = volume (mm³), t = time (min), k = the evaporation rate (mm/min), and A = surface area (mm2). Use finite difference method to compute the volume numerically of the droplet from t = 0 to 10 min using a step size of 1 min. Assume that k = 0.08 mm/min and that the droplet initially has a radius of 2.5 mm.

(b) Explain truncation error with help of a suitable example.

8 + 4 = 12

3. (a) With example discuss relative approximate error. Also write the difference between true error and approximate error.

(b) Consider three vectors *A = 2i - 3j + ak, B = bi + j - 4k, C = 3i + cj + 2k*
Now *A.B = A.C = 0* and *B.C = 2*. Use the LU decomposition method to find the values of the coefficients a, b and c with four significant digits.

(2 + 2) + 8 = 12

Group – C

4. (a) Use the Gauss-Seidel method to solve the following system to a tolerance of ε_s = 5%. If necessary, rearrange the equations to achieve convergence.

$$2x - 6y - z = -38$$
$$-3x - y + 7z = -34$$
$$-8x + y - 2z = -20$$

(b) Use least square regression to fit a straight line to the following data where y is f(x).

X	1	2	3	4	5	6	7
Y	1	1.5	2	3	4	5	8

7 + 5 = 12

5. (a) The following data define the sea-level concentration of dissolved oxygen for fresh water as a function of temperature.

T (⁰C)	0	8	16	24	32	40
C (mg/L)	14.621	11.843	9.870	8.418	7.305	6.413

Estimate the concentration of dissolved oxygen at 27°C using Lagrange interpolating polynomials.

(b) What is the convergence criterion for Gauss Seidel iteration method to solve the system of linear equations? Explain with an example.

8 + 4 = 12

Group – D

6. (a) Evaluate the definite integral of following function by two point Gauss quadrature formula from the limit 0 to 8.
$f(x) = -0.055x^4 + 0.86x^3 - 4.2x^2 + 6.3x + 2$

(b) Use Trapezoidal rule to evaluate the definite integral of following function from limit 0 to $\pi/2$ with n = 4.
f(x) = sin x + cos x

8 + 4 = 12

7. (a) When iterative methods are used to solve the linear system of algebraic equations, under what conditions convergence to the exact solution is guaranteed?

(b) Determine the largest eigenvalue in magnitude, and the corresponding eigenvector of the following matrix by power method. Use suitable initial approximation to the eigenvector.

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

2 + 10 = 12