(b) For which value of λ, the game with the following pay-off matrix is strictly determinable?

	PLAYER B		
PLAYER A	λ	6	2
	−1	λ	−7
	−2	4	λ

8 + 4 = 12

7. (a) Use dominance principle to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of it:

	PLAYER B		
PLAYER A	1	7	2
	6	2	7
	5	1	6

(b) Use graphical method in solving the following game and find the value of the game.

	PLAYER B	
PLAYER A	2	4
	2	3
	3	2
	−2	6

6 + 6 = 12

Group – E

8. (a) Find the extreme point(s) of the function $f(x_1, x_2, x_3) = -x_1^2 - 2x_2 - x_3^2 - 2x_1x_2$ and determine their nature.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:

Maximize $f(x_1, x_2) = 2x_1^2 - 3x_2^2 + 18x_2$

Subject to the constraints:

$$2x_1 + x_2 = 8$$

$$x_1, x_2 \geq 0$$

4 + 8 = 12

9. Maximize $z = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$

Subject to the constraints

$$x_2 \leq 8$$
$$x_1 + x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

by applying Kuhn-Tucker conditions.

12

OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The basic feasible solutions of the system of equations $x_1 + x_2 + x_3 = 8$ and $3x_1 + 2x_2 = 18$ are
(a) (2,6,0) and (6,0,2) (b) (1,7,0) and (7,1,0)
(c) no basic solution (d) (9,2,0) and (2,9,0).

(ii) To standardize an L. P. P. with " $\leq$ " type constraints, which variables are introduced
(a) slack (b) surplus (c) artificial (d) unrestricted.

(iii) In a transportation problem of size $m \times n$, a feasible solution is called a basic feasible solution if the number of non-negative allocation is equal to
(a) $m - n + 1$ (b) $m - n - 1$ (c) $m + n - 1$ (d) $m + n$.

(iv) The solution of a transportation problem is never
(a) unbounded (b) feasible (c) optimal (d) basic feasible.

(v) An assignment problem can be solved by
(a) VAM (b) matrix minima method
(c) Dominance principle (d) Hungarian method.

(vi) In a fair game the value of the game is
(a) 1 (b) 0 (c) 2 (d) unbounded.

(vii) The value of the game having the following pay-off matrix is

	PLAYER B		
PLAYER A	10	2	3
	7	6	8
	0	3	1

(a) 6 (b) 10 (c) 3 (d) 2.

(viii) In an assignment problem, the least possible number of lines required to cover all the zeroes of the reduced cost matrix of order n, for an optimal solution is
(a) n (b) $n-1$ (c) $n+1$ (d) n^2.

(ix) If $(1,1)$ is a stationary point of the function $f(x, y)$, which of the following condition assures that $(1,1)$ is a global minimum of it?

(a) $Hf(1,1)$ is negative definite

(b) $Hf(x, y)$ is negative definite for all (x, y)

(c) $Hf(x, y)$ is positive definite for all (x, y)

(d) $Hf(1,1)$ is positive definite.

(x) The quadratic form $Q(x, y) = -x_1^2 - 2x_2^2$ is

(a) positive definite (b) positive semi-definite

(c) negative definite (d) negative semi-definite.

Group – B

2. (a) Solve the following L.P.P. by graphical method:

Minimize $z = 20x_1 + 10x_2$

Subject to the constraints

$$x_1 + x_2 \le 40$$
$$3x_1 + x_2 \ge 30$$
$$4x_1 + 3x_2 \ge 60$$
$$x_1, x_2 \ge 0.$$

(b) Use Simplex method to solve the following linear programming problem:

Maximize $z = x_1 + x_2 + 3x_3$

Subject to $3x_1 + 2x_2 + x_3 \le 3$

$2x_1 + x_2 + 2x_3 \le 2$

$x_1, x_2, x_3 \ge 0.$

5 + 7 = 12

3. (a) Use 'Big-M' method to solve the following L.P.P.:

Maximize $z = 3x_1 - x_2$

Subject to the constraints

$$-x_1 + 3x_2 \ge 2$$
$$5x_1 - 2x_2 \ge 2$$
$$x_1, x_2 \ge 0.$$

(b) Write the dual of the following L.P.P.:

Minimize $z = 3x_1 - 2x_2$

Subject to $2x_1 + x_2 \le 1$

$-x_1 + 3x_2 \ge 4$

$x_1, x_2 \ge 0$

8 + 4 = 12

Group – C

4. (a) Find out the minimum cost for the assignment problem whose cost coefficients are follows:

	A	B	C	D	E
I	-2	-4	0	-6	-1
II	0	-9	-5	-5	-4
III	-3	-8	-9	-2	-6
IV	-4	-3	-1	0	-3
V	-9	-5	-8	-9	-5

(b) Solve the following transportation problem by matrix minima method:

	D_1	D_2	D_3	D_4	Availability
S_1	3	5	7	6	50
S_2	2	5	8	2	75
S_3	3	6	9	2	25
Demand	20	20	50	60	

8 + 4 = 12

5. (a) Find the optimum transportation schedule and minimum total cost of the following transportation problem:

	D_1	D_2	D_3	a_i
O_1	10	7	8	45
O_2	15	12	9	15
O_3	7	8	12	40
b_j	25	55	20	

(b) Formulate the following transportation problem as an LP model to minimize the total transportation cost.

	D_1	D_2	Availability (a_i)
S_1	19	30	7
S_2	70	30	9
Demand (b_j)	6	10	

8 + 4 = 12

Group – D

6. (a) Use algebraic method to solve the following game:

	PLAYER B		
PLAYER A	-1	2	1
	1	-2	2
	3	4	-3