9. (a) State Kuratowski's theorem.

(b) Obtain the dual of $k_4$ when embedded on the plane.

(c) Prove that the graph $k_{m,n}$ has a perfect matching if and only if m = n.

**2 + 4 + 6 = 12**





# DISCRETE MATHEMATICS
## (INFO 3133)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
## (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $p \leftrightarrow q \equiv (p \rightarrow q) \wedge r$ then $r$ is
(a) $p \rightarrow q$ (b) $\sim p$ (c) $q \rightarrow p$ (d) $\sim q$.

(ii) The contrapositive of $\sim p \rightarrow q$ is
(a) $p \rightarrow q$ (b) $\sim q \rightarrow \sim p$ (c) $\sim q \rightarrow p$ (d) $q \rightarrow \sim p$.

(iii) Everybody in a room shakes hands with everybody else. The total number of handshakes is 66. The total number of persons in the room is
(a) 11 (b) 12 (c) 13 (d) 14.

(iv) Determine which of the following statements is *not* true.
(a) The Petersen graph is non-planar
(b) $K_5$ is non-planar
(c) $K_{3,3}$ is non-planar
(d) The dual graph of a planar graph is non-planar.

(v) For any graph $G$, if $\omega(G)$ is the clique number of $G$ and if $\chi(G)$ is the chromatic number of $G$, then
(a) $\omega(G) = \chi(G)$ (b) $\omega(G) = \chi(G) + 3$
(c) $\omega(G) \leq \chi(G)$ (d) $\omega(G) \geq \chi(G)$.

(vi) The sum of the co-efficient in the expansion $(x + Y + z)^{10}$ is
(a) $2^{10}$ (b) $2^9$ (c) $3^{10}$ (d) $3^9$.

(vii) The chromatic polynomial of a tree with $n$ vertices is
(a) $\lambda^{n-1}(\lambda-1)$ (b) $\lambda^{n}(\lambda-1)^{n}$ (c) $\lambda(\lambda-1)^{n-1}$ (d) $\lambda(\lambda-1)^{n}$.

(viii) The matching number for the graph $K_{n,n}$ is
(a) 2 (b) $n$ (c) $n^2$ (d) $n - 1$.

(ix) Negation of $\exists x \forall y, P(x,y)$ is
(a) $\forall x\ \forall y, \sim P(x,y)$ (b) $\forall x\ \exists y, \sim P(x,y)$
(c) $\exists x\ \exists y, \sim P(x,y)$ (d) $\forall x\ \forall y, P(x,y)$.

(x) A solution of $3x \equiv 1(mod\ 7)$ is x ≡
(a) 2 (mod 7) (b) 5 (mod 7)
(c) 4 (mod 7) (d) 6 (mod 7).

## Group – B

2.(a) Prove the following without constructing the truth table.
$(q \to (p \wedge \sim p)) \to\ (r \to (p \wedge \sim p)) \equiv r \to q$

(b) Determine which of the following compound proposition tautology is and which one is contingency.
i) $(\sim p \leftrightarrow \sim q) \leftrightarrow (q \leftrightarrow r)$.
ii) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

**4 + (4 + 4) =12**

3.(a) Obtain the principal disjunctive normal form and the principal conjunctive normal form of the following statement:
$p \vee (\sim p \to (q \vee (\sim q \to r)))$.

(b) Check the validity of the following argument.
"If a number is divisible by 6, then it is divisible by 3. The number is not divisible by 3. Therefore, the number is not divisible by 6."

**7 + 5 = 12**

## Group – C

4.(a) Show that $7^{402}+16! = 14(mod 17)$. Show your calculations in detail, and state every theorem that you use.

(b) Let $p$ be a prime number.
If $a \equiv b\ (mod\ p)$, then prove that $a^k \equiv b^k\ (mod\ p)$ for every positive integer $k$. Is it true that if $a^k \equiv b^k\ (mod\ p)$, then $a \equiv b\ (mod\ p)$? Justify your answer.

**6 + 6 = 12**



B.TECH/IT/5TH SEM/INFO 3133/2019

5.(a) Find the general solution of the equation $5x + 32y = 28$ in integers. (Express your solution in terms of a parameter $t$.)

(b) Prove that there is an infinite number of prime numbers.

**6 + 6 = 12**

## Group – D

6.(a) Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8 ?
i. How many of the numbers in part (i) are less than 4000 ?
ii. How many of the numbers in part (i) are even?
iii. How many of the numbers in part (i) are odd?
iv. How many of the numbers in part (i) are multiples of 5 ?
v. How many of the numbers in part (i) contain both the digits 3 and 5 ?

(b) How many positive integers less than 10, 00,000 have the sum of their digits equal to 19?

**(6 ×1)+ 6= 12**

7. (a) Five gentlemen $A, B, C, D$ and $E$ attend a party, where before joining the party, they leave their overcoats in a cloak room. After the party, the overcoats get mixed up and are returned to the gentlemen in a random manner. Using the principle of inclusion-exclusion, find the probability that none receives his own overcoat.

(b) Solve the recurrence relation
$a_n - 6a_{n-1} + 8a_{n-2} = 9$ for n≥ 2 where $a_0 = 10$ and $a_1 = 25$.

**6 + 6 = 12**

## Group – E

8. (a) In an examination seven subjects are to be scheduled; $S_1, S_2, S_3, S_4, S_5, S_6, S_7$. Following pairs of subjects have common students: $(S_1,S_2)$, $(S_1,S_3)$, $(S_1,S_4)$, $(S_1,S_7)$, $(S_2,S_3)$, $(S_2,S_4)$, $(S_2,S_5)$, $(S_2,S_7)$, $(S_3,S_4)$, $(S_3,S_6)$, $(S_3,S_7)$, $(S_4,S_5)$, $(S_4,S_6)$, $(S_5,S_6)$, $(S_5,S_7)$ and $(S_6,S_7)$. How can the examination be scheduled so that no student has two examination at the same day?

(b) Prove that $K_{3,3}$ is non-planar.

**6 + 6 = 12**