INFORMATION THEORY & CODING
(INFO 2111)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which is not a field element of the polynomial, $p(x) = x^5 + x^2 + 1$ in GF (2^6).
(a) $\alpha^3 + \alpha$ (b) $\alpha^4 + \alpha^2$ (c) $\alpha^4 + 1$ (d) $\alpha^3 + \alpha + 1$.

(ii) Channel capacity is exactly equal to –
(a) bandwidth of demand
(b) Amount of information per second
(c) Noise rate in the demand
(d) None of the above.

(iii) For M equally likely messages, the average amount of information H is
(a) $H = \log_{10}M$ (b) $H = \log_2 M$ (c) $H = \log_{10}M^2$ (d) $H = 2\log_{10}M$.

(iv) The information rate R for given average information H= 2.0 for analog signal band limited to B Hz is
(a) 8 B bits/sec (b) 4 B bits/sec (c) 2 B bits/sec (d) 16 B bits/sec.

(v) Which of the following code is a class of non-binary BCH?
(a) Hamming code (b) Hadamard code
(c) Golay code (d) Reed Solomon codes.

(vi) The relation between entropy and mutual information is
(a) I(X;Y) = H(X) - H(X/Y) (b) I(X;Y) = H(X/Y) - H(Y/X)
(c) I(X;Y) = H(X) - H(Y) (d) I(X;Y) = H(Y) - H(X).

(vii) The memory less source refers to
(a) no previous information
(b) no message storage
(c) emitted message is independent of previous message
(d) none of the above

(viii) For the generation of a cyclic code, the generator polynomial should be the factor of ____
(a) $x^n + 1$ (b) $x^n - 1$ (c) $x^n / 2$ (d) $x^{2n/3}$.

(ix) dfree is defined as the Euclidean distance of coded signal in terms of _____ possible distance between all allowed sequences.
(a) smallest (b) largest (c) average (d) constant.

(x) For BCH code if the received vector and the computed vector are r(x) and e(x) respectively, then the error free code vector is_____.
(a) r(x) *e(x) (b) r(x)/e(x) (c) r(x) + e(x) (d) None of these.

Group – B

2. (a) The International Morse Code uses a sequence of symbols of dots and dashes to transmit letters of English alphabet. The dash is represented by a current pulse of duration 2 ms and dot by a duration of 1 ms. The probability of dash is half as that of dot. Consider 1 ms duration of gap is given between the symbols. Calculate:

i. Self-information of a dot and a dash
ii. Average information content of a dot-dash code
iii. Average rate of information

(b) Consider a system emitting one of the three symbols A, B and C with respective probabilities 0.7, 0.15 and 0.15. Calculate its efficiency and redundancy.

(c) Proof I(X;Y)=I(Y;X)

(3 × 2) + 3 + 3 = 12

3. Consider a discrete memoryless source with S={X, Y, Z} with the state probabilities P={0.7, 0.15, 0.15} for its output.
(i) Apply Huffman Encoding Algorithm to find the code-words in binary. Find the source efficiency and redundancy.
(ii) Consider the second-order extension of the source. Compute the code-word for this extended source and also find its efficiency.

(6 + 6) =12

Group – C

4. Consider a (8,4) Hamming Code whose parity check equations are:
$C_1 = m_1 \oplus m_2 \oplus m_3 \oplus m_4$
$C_2 = m_1 \oplus m_3 \oplus m_4$
$C_3 = m_1 \oplus m_4$
$C_4 = m_2 \oplus m_3 \oplus m_4$
Find out
(i) Generator Matrix G
(ii) Parity Check Matrix H
(iii) d_{min} distance
(iv) How many errors it can detect and correct?

(2+2+5+3)=12

5. (a) Divisor polynomial of CRC is x^3+x+1. Find out the sent code-word of message 1 1 0 1 0 1 1 0.

(b) Find out the position of error bit of received message 1 1 0 1 0 1 1 0 1 0 0 1 0 using Hamming Code.

(6 + 6) =12

Group – D

6. (a) Construct the field GF(2^5) for the given polynomial $p(x) = x^5 + x^2 + 1$.

(b) Find (i) $\alpha^5 + \alpha^{12} + \alpha^{14}$ (ii) $\alpha^3 + \alpha^7 + \alpha^{11}$ (iii) $\alpha^{11} + \alpha^{13} + \alpha^2$ in GF (2^4).

(c) Show that α^5 is a primitive element of GF(2^3).

(3 + 6 + 3)=12

7. (a) Find the generator polynomial g(x) for a double error correcting binary BCH code of block length 15 over GF (16). Use primitive polynomial $p(x) = x^4+x+1$.

(b) Find the Minimal Polynomial for the field element α^3 in GF (2^5). Use primitive polynomial $p(x) = x^5 + x^2 + 1$ to construct GF (2^5).

7 + 5 = 12

Group – E

8. A rate 1/3 convolutional coder with consistent length of '3' uses for generating vectors
g_1=(1 0 0), g_2=(1 1 1) and g_3=(1 0 1)
(i) Sketch the encoder configuration.
(ii) Draw the code tree (up to three levels), state diagram and Trellis diagram.
(iii) If input message sequence is 10110, determine the output sequence of the encoder

2 + (4 +2 + 2) + 2=12

9. A (3, 1, 3) convolutional coding is generating impulse responses
g_1=(1 0 1 0), g_2=(1 0 0 1) and g_3=(1 1 1 1)
Encode the message 'M={1 0 0 1 0 1}' using time domain and transfer domain approaches.

(6 + 6) =12