Group – E

8. (a) Check the validity of the following argument:

"If I pass B. Tech. with high YGPA, I will be assured of a good job. If I am assured of a good job then my parents will be happy. My parents are not happy. Therefore, I do not pass B. Tech. with high YGPA."

(b) Show that the statement formula $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is a tautology after expressing it in conjunctive normal form.

6 + 6 =12

9. (a) Consider the following:
p: Question paper is hard
q: I shall fail in the examination

Translate the following into symbols:
(i) If question paper is hard, then I shall fail in the examination
(ii) If I pass in the examination, then question paper is not hard
(iii) Question paper is not hard if and only if I pass in the examination
(iv) If question paper is not hard then I shall pass in the examination

(b) Without using truth table establish the equivalence for the following function:
$(p \wedge q) \vee (\sim p \vee (\sim p \vee q)) = \sim p \vee q$.

(c) Find the principal disjunctive normal form of $\sim p \wedge (q \leftrightarrow p)$.

4+ 4 + 4 = 12

DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If C_{97} be a cycle with 97 vertices then $\chi(C_{97})$ =
(a) 97 (b) 98 (c) 2 (d) 3.

(ii) $((p \rightarrow q) \rightarrow p) \rightarrow q$ =
(a) p (b) q (c) $q \rightarrow p$ (d) $p \rightarrow q$.

(iii) For every integer x, gcd($x, x + 2$) =
(a) 0 (b) 2 (c) 1 (d) (b) or, (c).

(iv) Determine which one of the following is true:
(a) $50 \equiv 13 (\text{mod } 7)$ (b) $-9^3 \equiv 7 (\text{mod } 8)$
(c) $123 \equiv 13 (\text{mod } 9)$ (d) $8^4 \equiv 2 (\text{mod } 13)$.

(v) The generating function for the sequence $\{x_n\}$, where, x_n=3, for all n≥0 is
(a) 1/(1-x) (b) 1/(1+x) (c) 3/(1-x) (d) 3/(1+x).

(vi) What is the chromatic number of an n-vertex simple connected graph (n≥2) which does not contain any cycle of odd length?
(a) 2 (b) 3 (c) n-1 (d) n.

(vii) Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is
(a) 6 (b) 8 (c) 9 (d) 13.

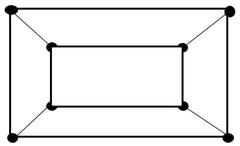
(viii) If a divides c and b divides c with gcd(a,b)=1, then
(a) c divides ab (b) ab divides c
(c) gcd (a,c)=1 (d) gcd(a,c)=2.

(ix) The remainder in the division of 1!+2!+3!+.....+1000! by 4 is
(a) 0 (b) 1 (c) 9 (d) 2.

(x) Let p be the proposition "it is cold" and q be the proposition" it is raining". Then the symbolic form of the statement "it is cold and it is raining" is
(a) $p \vee q$ (b) $\sim p \vee q$ (c) $p \wedge q$ (d) $p \vee \sim q$

Group – B

2. (a) Define matching number. Find the matching number for the following graph:



(b) Deduce the number of perfect matchings in K_n.

(c) Let G be a planar graph with n vertices, e edges, f faces and k connected components, then prove that n-e+f=k+1.

2 + 4 + 6 = 12

3. (a) State Kuratowski's theorem. Apply the same to check the planarity of the graph given in Fig. 1.

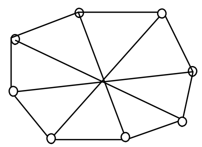


Fig. 1.

(b) Applicants a_1, a_2, a_3 and a_4 apply for five posts p_1, p_2, p_3, p_4 and p_5. The applications are done as follows : $a_1 \to \{p_1, p_2\}, a_2 \to \{p_1, p_3, p_5\}, a_3 \to \{p_1, p_2, p_3, p_5\}$ and $a_4 \to \{p_3, p_4\}$. Using Hall's marriage theorem find whether there is a perfect matching of the set of applicants into the set of objects?
If yes, obtain the matching.

7 + 5 =12

Group – C

4. (a) Use the Principle of Finite Induction to show that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2}{3}n^3 - \frac{n}{105}$ is an integer $\forall n \in \mathbb{N}$.

(b) Let $a > 1$ and m, n be positive integers. Then show that $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m,n)} - 1$.

(c) If a= qb+r, then prove that gcd (a,b)=gcd(b,r), where a,b,q and r are positive integers.

3+ 3 + 6 = 12

5. (a) Solve the following system of linear congruences:
$x \equiv 4 \bmod (12)$
$x \equiv 7 \bmod (21)$
$x \equiv 10 \bmod (15)$

(b) If gcd(a,b)=1, then show that $b|ap \Rightarrow b|p, \forall a, b, p \in \mathbb{Z}$.

(c) Obtain the remainder when $6^{17} + 17^6$ is divided by 7.

6+ 3 + 3 = 12

Group – D

6. (a) Each user of a computer system has a password, which is six to eight characters long, where, each character is an uppercase or, a lowercase English letter or, a digit or, a special character from the set {#,∗, $}. Each password must contain at least one letter and one digit and one special character. How may possible passwords are there?

(b) Determine the coefficient of x^8y^5 in the expansion of $(x + y)^{13}$.

(c) State and prove the generalized Pigeonhole Principle.

4+ 2 + (2 + 4) = 12

7. (a) Find the generating function for the sequence $\{n(7 + 2n)\}, n \in \mathbb{N}$.

(b) Solve the recurrence relation,
$f_n = f_{n-1} + f_{n-2}$, n≥3.

6 + 6 = 12