6 + 6 = 12

Group – D

6.(a) Find the extreme point(s) of the function
$f(x,y,z) = x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 16yz$
and determine their nature also.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:
Minimize $Z = (x-3)^2 + (y+1)^2 + (z-2)^2$
Subject to the constraints
$3x - 2y + 4z = 9$
$x + 2y = 3$

12

7. Minimize $z = 2x_1 + 3x_2 - x_1^2 - 2x_2^2$

Subject to the constraints
$x_1 + 3x_2 \leq 6$
$5x_1 + 2x_2 \geq 10$
$x_1, x_2 \geq 0$

by applying Kuhn-Tucker conditions.

12

Group – E

8. Find the maximum of $f(x) = 2x - 1.75x^2 + 1.1x^3 - 0.25x^4$ using Golden Section Search algorithm over $[-2, 4]$ with a tolerance limit of 1%.

12

9. Write the Golden Section Search Algorithm for unimodal functions of one variable and using the algorithm maximize $f(x) = -x^2 2x$ over $-3 \leq x \leq 6$ with a tolerance to be less than 0.2.

12

OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES
(MATH 4181)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The set S given by S={(x,y): $x^2+y^2 \leq 25$} is
(a) convex (b) open (c) non-convex (d) unbounded.

(ii) If in the Big-M method, the set of basic variables of the final simplex table contains an artificial variable, the problem has
(a) degenerate solution (b) infeasible solution
(c) unbounded solution (d) multiple optimal solution.

(iii) An LPP with '≥' type constraints can be solved by using
(a) Simplex Method (b) Graphical Method.
(c) Big-M Method. (d) North West Corner Rule.

(iv) The feasible region of an L.P.P. is
(a) convex (b) non-convex
(c) open (d) unbounded.

(v) Assignment problem is solved by
(a) North West Corner Rule.
(b) Simplex Method.
(c) Vogel's Approximation Method.
(d) Hungarian Method.

(vi) Which of the following is an algorithm to solve an assignment problem
(a) Fibonacci search (b) Golden section search
(c) Dichotomous search (d) Hungarian method.

(vii) The basic solution to a system of linear simultaneous equations with four equations and five variables would assign a value 0 to
(a) 4 variables. (b) 2 variables.
(c) 1 variable. (d) None of the variables.

(viii) The function $f(x, y) = 2xy - x^4 - x^2 - y^2$ has, at $(0, 0)$,
(a) a global minimum point. (b) a saddle point.
(c) a global maximum point. (d) a local minimum point.

(ix) The Hessian matrix of function $f(x, y, z)$ is

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

If the function had a stationary point, this would be
(a) a local maximum point. (b) a global minimum point.
(c) a saddle point. (d) a global maximum point.

(x) Given the optimization problem
minimize $f(x, y)$
subject to $3x - 6y = 9$
If $(x, y, \lambda) = (1, -1, 3)$ is a stationary point of the associated Lagrange function, it can be assured that $(1, -1)$ is a global minimum of the problem when the function $f(x, y)$ is
(a) convex. (b) non-convex.
(c) concave. (d) neither convex nor concave.

Group – B

2. (a) Solve the following L.P.P. by graphical method:
Maximize $z = 5x_1 + 3x_2$
Subject to the constraints
$$4x_1 - 3x_2 \le 12$$
$$-x_1 + x_2 \ge -2$$
$$3x_1 + 2x_2 \ge 12$$
$$x_1 \ge 2$$
$$0 \le x_2 \le 4.$$

(b) Solve the following L.P.P. by Simplex method:
Maximize $z = 2x_1 + x_2$
Subject to the constraints
$$2x_1 - x_2 \le 8$$
$$2x_1 + x_2 \le 12$$
$$-x_1 + x_2 \le 3$$
$$x_1,\ x_2 \ge 0.$$

4 + 8 = 12

3. (a) Use Charne's Big M method to solve the following linear programming problem:
Minimize $Z = 25x_1 + 15x_2 + 5x_3$
Subject to the constraints:
$4x_1 + 2x_2 + x_3 \ge 10$
$2x_1 + 3x_2 \ge 6$
$3x_1 + x_3 \ge 8$
$x_1, x_2, x_3 \ge 0$

(b) Write down the dual of the following linear programming problem:
Minimize $Z = 4x_1 + 3x_2 - 6x_3$
Subject to the constraints:
$x_1 - x_3 \ge 2$
$x_2 - x_3 \ge 5$
$x_1, x_2 \ge 0$ and x_3 is unrestricted in sign

9 + 3=12

Group – C

4. (a) Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method:

	W1	W2	W3	W4	Capacity
F1	10	30	50	10	7
F2	70	30	40	60	9
F3	40	8	70	20	18
Requirement	5	8	7	14	

(b) A salesman has to visit five cities A, B, C, D and E. The distance (in hundred miles) between the five cities are as follows:

	A	B	C	D	E
A	-	7	6	8	4
B	7	-	8	5	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A, which route should he select so that the total distance travelled is minimum?

6 + 6=12

5. (a) Use graphical method to solve the game with following pay-off matrix and find the value of the game.

PLAYER B

2	2	3	−2	PLAYER A
4	3	2	6	

(b) Players A and B play a game in which each has three coins, a 2 Rs., 5 Rs. and a 10 Rs. Each selects a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, then A wins B's coin. But, if the sum is even, then B win's A's coin. Find the best strategy for each player and the values of the game by dominance principal.