7. (a) Given $x(n) = a^n u(n)$ Find the
(i) Fourier transform (ii) z-transform and ROC.

(b) Sketch the sequence x(n)= $\sum_{k=-\infty}^{\infty} \delta(n-3k)$ and find its discrete Fourier series.

(3 + 4) +5 = 12

Group – E

8. (a) (i) What is meant by Autocorrelation function?
(ii) Explain the relation between Autocorrelation function and Energy Spectral Density function of a signal.

(b) Consider the analog signal given by x(t) = 10sin 50πt sin 300πt
(i) What is the Nyquist rate of this signal?
(ii) State the problems if the signal is sampled at its Nyquist rate?
(iii) How do we overcome the previous problem?

(2 + 4) + 6 = 12

9. (a) How do you define the following terms in the light of random process:
(i) Sample space (ii) Mean (iii) Correlation (iv) Ergodic process.

(b) Explain briefly the reconstruction of sampled signals at the receiver end of a communication system.

(2 × 4) + 4 = 12

SIGNALS AND SYSTEMS
(ECEN 2103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The discrete time x(n)=(-1)^n is periodic with fundamental period :
(a) 1 (b) 2
(c) ∞ (d) none of the above.

(ii) The time derivative of unit step function is
(a) a unit impulse (b) a step function
(c) a ramp function (d) a sine function.

(iii) A system whose output depend on future inputs is a
(a) static system (b) dynamic system
(c) non-causal system (d) both (b) and (c).

(iv) The area under the Fourier transform, i.e. $\int_{-\infty}^{\infty} X(\omega)d\omega =$
(a) x(0) (b) X(0)
(c) 2πx(0) (d) (1/2π)x(0).

(v) The area under the curve $\int_{-\infty}^{\infty} \delta(t)dt$ is
(a) infinity (b) unity
(c) zero (d) undefined.

(vi) Fourier Transform of a d.c signal with unity strength is
(a) zero (b) 1
(c) $2\pi\delta(\omega)$ (d) $2\delta(\omega)$.

(vii) A signal is band-limited to 50 kHz. The signal can be uniquely determined by its values at uniform intervals less than
(a) 50 μs (b) 50 ms
(c) 100 μs (d) 10 μs.

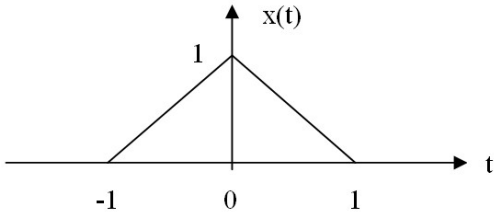
(viii) If a periodic signal has odd symmetry, then the Fourier series has
(a) only sine term (b) only cosine term
(c) both sine & cosine terms (d) none of the above.

(ix) Region of Convergence of a causal LTI system
(a) is entire s-plane (b) is right half of s-plane
(c) is left half of s-plane (d) does not exist.

(x) The signum function can be written as follows:
(a) u(t)-1 (b) 2u(t)-1
(c) 1-u(t) (d) u(t)+u(-t).

Group – B

2. (a) Perform the operation x(-2t-1) for the following function x(t).

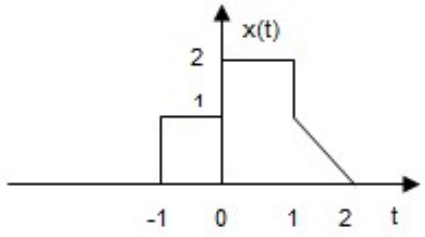


(b) Plot the function $x(t) = u(t) - 2u(t-1) + u(t-2)$. Also plot the derivative, $\frac{dx}{dt}$ of the function.

(c) Find and sketch the even and odd components of the function, $x(t) = tu(t)$.

4 + 4 + 4 = 12

3. (a) Given a signal x(t) as follows



Draw x(3t+2).

(b) Given an LTI system described by the differential equation $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y = \frac{dx(t)}{dt} + 4x(t)$. The input is $x(t)=e^{-t}\,u(t)$ and the initial conditions are $y(0^+)=3$, $\frac{dy(0^+)}{dt} = 0$. Find the natural response and forced response of the system.

(c) Given a system defined by its impulse response $h(t)=\frac{1}{RC}e^{-t/RC}u(t)$. Comment on the stability of the system.

3 + 7 + 2 = 12

Group – C

4. (a) Determine whether the system defined by the impulse response $h(t) = e^{-6|t|}$ is stable or not.

(b) Using Property of Fourier transform evaluate the Fourier transform of $x(t) = t\cos 2t$.

(c) Using Fourier transform find the convolution of the signals $x_1(t) = e^{-at}u(t)$ and $x_2(t) = e^{-bt}u(t)$.

3 + 4 + 5 = 12

5. (a) Find the transfer function H(s) of the system described by the following differential equation $\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t) = 5\frac{dx(t)}{dt} + 3x(t)$.

(b) Find the step response of a system whose impulse response is given by $h(t) = \delta(t) - \delta(t-1)$.

(c) Determine the Laplace Transform of a ramp function and state its Region Of Convergence.

4 + 4 + 4 = 12

Group – D

6. (a) Consider a causal and stable LTI system that is characterized by the difference equation $y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$. Find the frequency response $H(e^{jw})$ and the impulse response h(n) of the system.

(b) If X(z) is given by $X(z) = \frac{z+1}{3(z-1)(z+0.9)}$ Find the steady state value of x(n) if it exists.

(c) Determine the inverse z-transform for X(z) given as $X(z) = \frac{z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$, ROC: $2 < |z| < 3$.

6 + 3 + 3 = 12