6 + 6 = 12





# MATHEMATICAL METHODS
## (MATH 2001)

**Time Allotted : 3 hrs** **Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

## Group – A
## (Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of the complex integral $\int_c \frac{3z^2+7z+1}{z+1}dz$ where, $C:|z|=\frac{1}{3}$ is

(a) $2\pi i$ (b) 0 (c) $\frac{\pi i}{2}$ (d) $\pi i$.

(ii) The points of singularities of the ordinary differential equation $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ are

(a) 1, 0 (b) −1, 0 (c) 1, −1 (d) 0, 2 .

(iii) The value of $a_0$ in the Fourier series of $f(x)=2x-3x^2$ in $(0,3)$ is

(a) −6 (b) −12 (c) 6 (d) 12.

(iv) The solution of $pq + p + q = 0$ (where, $\equiv \frac{\partial}{\partial x}$, $q \equiv \frac{\partial}{\partial y}$) is

(a) $z = ax - \frac{a}{a+1}y + c$ (b) $z = ax + \frac{a}{a+1}y + c$

(c) $z = ax + a^2y + c$ (d) $z = ax + \frac{a}{a+1}x^2 + c$ .

(v) If $F(s)$ be the Fourier transform of $f(t)$, then the Fourier transform of $tf(t)$ is

(a) $\frac{d}{ds}\{F(s)\}$ (b) $i\frac{d}{ds}\{F(s)\}$

(c) $-i\frac{d}{ds}\{F(s)\}$ (d) $-\frac{d}{ds}\{F(s)\}$ .

(vi) $J_{1/2}(x)$ is

(a) $\sqrt{\frac{2\pi}{x}}\sin x$ (b) $\sqrt{\frac{2\pi}{x}}\cos x$ (c) $\sqrt{\frac{\pi}{2x}}\cos x$ (d) $\sqrt{\frac{2}{\pi x}}\sin x$.

(vii) The value of $\int_{-1}^{1} P_2(x)\,P_5(x)dx$ is

(a) 0 (b) 1 (c) 2 (d) 3.

(viii) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is

(a) $z = f_1(y+x) + f_1(y-x)$ (b) $z = f_1(y+x) + f_2(y-x)$
(c) $z = f_2(y+x) + f_2(y-x)$ (d) $z = f(x^2 - y^2)$.

(ix) Bessel's equation of order zero is

(a) $xy'' + y' - xy = 0$ (b) $xy'' + y' = 0$
(c) $xy'' - y' + xy = 0$ (d) $xy'' + y' + xy = 0$.

(x) For $f(z)=\frac{z-\sin z}{z^3}$, the point $z=0$ is

(a) a pole of order 3 (b) a pole of order 2
(c) a simple pole (d) a removal singularity.

## Group – B

2. (a) Evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)}\,dz$ where $C$ is the circle $|z| = 2$, using Cauchy's residue theorem.

(b) State Taylor's series. Expand $\sin z$ and $\cos z$ in Taylor's series about $z = 0$.

**6 + 6 = 12**

3. (a) (i) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in Laurent's series valid for $1<|z|<2$.

(ii) Expand $f(z)=\frac{\sin z}{z-\pi}$ about $z = \pi$.

(b) Construct an analytic function $f(z)=u(x,y)+iv(x,y)$
Where, $v(x,y)=6xy-5x+3$. Express the function $f(z)$ as a function of $z$.

**(3 + 3) + 6 =12**



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## Group – C

4. (a) Find the Fourier series expansion of the function $f(x) = x\cos x$ in $(-\pi,\pi)$.

(b) Using Parseval's identity, prove that $\int_0^\infty \frac{dt}{(4+t^2)(25+t^2)} = \frac{\pi}{140}$.

**6 + 6 = 12**

5. (a) Find the Fourier sine series of the function $f(x)=x(\pi-x)$ in $0\le x\le\pi$.
Hence prove that $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots=\frac{\pi^3}{32}$.

(b) Find $f(x)$ if its Fourier sine transform is $\frac{1}{s}e^{-as}$. Hence deduce $F_s^{-1}\left(\frac{1}{s}\right)=1$.

**6 + 6 = 12**

## Group – D

6. (a) Obtain the power series solution of $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+x^2y=0$ about x = 0.

(b) Express $x^3+x^2$ in terms of Legendre polynomial $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$.

**7 + 5 = 12**

7. (a) Prove: $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$.

(b) Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

**6 + 6 = 12**

## Group – E

8. (a) Form the partial differential equation by eliminating the arbitrary function $f$ from $f(x^2+y^2, z-xy) = 0$.

(b) Solve by Lagrange's method: $y^2p - xyq = x(z-2y)$.

**6 + 6 = 12**

9. (a) Find the general solution of the following partial differential equation
$(D^2-DD'-2D'^2)z=(y-1)e^x$, (where, $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$)

(b) Solve : $2z + p^2 + qy + 2y^2 = 0$.