

Group - E

8. (a) Using Taylor's series method solve the differential equation:
 $\frac{dy}{dx} = -2xy^2$ where, $y(0) = 1$. Calculate, $y(0.2)$ and $y(0.4)$ with step length 0.2.

- (b) Fit a second degree polynomial from the following data from Table 5:

x	1	2	3	4	5
y	3	7	13	21	31

Table 5

6 + 6 = 12

9. (a) Given, $\frac{dy}{dx} = y - x$, $y(0) = 2$. Applying fourth order Runge-Kutta method Find $y(0.1)$ and $y(0.2)$ correct to four decimal places.

- (b) Fit a second degree parabola $y = ax^2 + bx + c$ with the following Table 6.

x	2	3	5	6	8
y	11.7	25.7	72.5	105.5	190.7

Table 6

6 + 6 = 12
NUMERICAL ANALYSIS
(MCAP 1103)
Time Allotted : 3 hrs**Full Marks : 70***Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group - A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) In trapezoidal rule the error is zero if $f(x)$ is of degree
 (a) 1 (b) 2
 (c) 3 (d) any of (a), (b) and (c).
- (ii) Euler's method is a _____ order Runge-Kutta method.
 (a) first (b) second
 (c) fourth (d) third
- (iii) The convergency criterion of direct iterative method $x = \phi(x)$ is
 (a) $|\phi'(x)| < 1$ (b) $|\phi(x)| > 1$
 (c) $|\phi(x)| = 1$ (d) none of (a), (b) and (c).
- (iv) The number of significant digits in 1.00234 is
 (a) 4 (b) 6
 (c) 3 (d) 5.
- (v) The values of λ for which the equation $Ax = \lambda x$ has a non-trivial solutions are called
 (a) Eigen values (b) Eigen vectors
 (c) Trivial values (d) Inverse of A.
- (vi) If a be the actual value and e be it's estimated value, then the relative error is
 (a) $\frac{a}{e}$ (b) $\frac{|a-e|}{e}$
 (c) $\frac{a-e}{e}$ (d) $\frac{|a-e|}{a}$.
- (vii) Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If iterations are started from - 1, then iterations will be
 (a) converge to -1 (b) converge to $\sqrt{2}$
 (c) converge to $-\sqrt{2}$ (d) No converge.

- (viii) The number of different polynomials that can go through two fixed data points (x_1, y_1) and (x_2, y_2) is
 (a) 0 (b) 1
 (c) 2 (d) infinite.
- (ix) The convergence of which of the following methods is sensitive to starting value?
 (a) False position (b) Gauss-Seidal method
 (c) Newton-Raphson method (d) All of (a), (b) and (c).
- (x) When $\Delta f(x) = f(x + h) - f(x)$, then constant $k = ?$
 (a) $f(x+k) - f(x)$ (b) $f(k) - f(0)$
 (c) 0 (d) 1.

Group - B

2. (a) Find a positive root of $xe^{-x} = 2$ by the method of false position.
 (b) Find a real root of $x^3 - x = 1$ between 1 and 2 by bisection method. Compute ten iterations.
6 + 6 = 12
3. (a) Find a real root of the equation $\cos x = 3x - 1$ correct to 3 decimal places using iteration method.
 (b) A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of the secant method.
6 + 6 = 12

Group - C

4. (a) Construct a backward difference table for $y = \log x$ given in Table 1 and find values of $\nabla^3 \log 40$ and $\nabla^4 \log 50$.

x	10	20	30	40	50
y	1	1.3010	1.4771	1.6021	1.6990

Table 1

- (b) The population of a town in the decadal census was as given Table 2. Estimate the population for the year 1895.

Year x	1891	1901	1911	1921	1931
Population y (in thousands)	46	66	81	93	101

Table 2

6 + 6 = 12

5. (a) Using Lagrange interpolation method compute the value of $f(x)$ for $x = 2.5$ from the following Table 3

X	1	2	3	4
Y	1	28	27	34

Table 3

- (b) Using Newton's divided difference formula, find a polynomial function satisfying the following data from table 4. Hence find $f(1)$.

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

table 4

6 + 6 = 12

Group - D

6. (a) The following relation gives the velocity v of a body during the time t specified. Find it's acceleration at $t = 1.15$.

t:	1.1	1.2	1.3	1.4	1.5
v:	43.1	47.7	52.1	56.4	60.8

- (b) Evaluate, $\int_0^5 \frac{dx}{4x+5}$ dividing the range into 10 equal parts with the help of Simpson's 1/3 rule. Then find an approximate value of $\log_e 5$ by calculating to 4 decimal places.
6 + 6 = 12

7. (a) What is Pivot element in a system of linear equation? What do you mean by partial pivoting? Solve the following system of linear equations using Gauss elimination method.

$$\begin{aligned} 2x + 3y - z &= 4 \\ 5x - 3y + 9z &= 11 \\ 2x + y + 4z &= 7 \end{aligned}$$

- (b) Solve the following system of linear equations by Gauss-Seidel method.

$$\begin{aligned} x + 2y + z &= 8 \\ 2x + 3y + 4z &= 20 \\ 4x + 3y + 2z &= 16 \end{aligned}$$

6 + 6 = 12