(b) Construct a Moore-machine equivalent to the Mealy-machine M given in Table 1.

Table 1

Present State	Next State			
	a=0		a=1	
	State	Output	State	Output
→Q	Q_1	1	Q_2	0
Q	Q_4	1	Q_4	1
Q	Q_2	1	Q_3	1
Q	Q_3	0	Q_1	1

6 + 6 = 12

MATHEMATICAL FOUNDATIONS
(MCAP 1102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following is a member of the set {{2,3},{2,4},{3,4}}
(a) 2 (b) 3 (c) 4 (d) none.

(ii) The number of vertices of odd degree in a graph G is always
(a) prime number (b) even number
(c) non prime number (d) odd number.

(iii) Find the rank of the word LETTER, when the letters are arranged as in dictionary.
(a) 13 (b) 14 (c) 15 (d) 16.

(iv) The generating function for the numeric function $(1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, -\frac{1}{6}, \ldots\ldots)$
(a) $\log(1+x)$ (b) e^x
(c) $\log(1+x)/x$ (d) $\log(1-x)/x$.

(v) Every vertex of a null graph is
(a) pendant (b) isolated
(c) odd (d) none of (a), (b) and (c).

(vi) The symmetric difference between non-empty sets is
(a) commutative (b) associative
(c) commutative but not associative (d) both (a) and (b).

(vii) $r \times n_{C_r} = n-1_{C_{r-1}} \times$ _________ .
(a) n - 1 (b) n
(c) r (d) r - 1

(viii) How many automobile license plates can be made if each plate contains two different letters followed by three different digits?
(a) 468000 (b) 676000
(c) 650000 (d) 486720.

(ix) If K pigeons are assigned to n pigeonholes, then one of the pigeonholes must contain at least ____________ pigeons.

(a) $\left\lfloor \frac{K}{n} \right\rfloor + 1$ (b) $\left\lceil \frac{K-1}{n} \right\rceil$

(c) $\left\lfloor \frac{K-1}{n} \right\rfloor + 1$ (d) $\left\lfloor \frac{K}{n} \right\rfloor - 1$

(x) If e is the number of edges, n is the number of vertices and k is the number of components of a graph then

(a) e<n-k (b) e≥n-k

(c) n<k (d) none of these.

Group – B

2. (a) A relation R defined on N by "mRn if m is a divisor of n for all l, m ϵN". Examine whether R is (i) reflexive (ii) symmetric (iii) transitive.

(b) Prove that (P∧ (P↔Q))→Q is a tautology.

(c) Prove that for any three sets A, B, C : A × (BUC) = (A × B) U (A × C).

(2 + 2 + 2) + 3 + 3 = 12

3. (a) Show that the set of all fourth root of unity forms a group with respect to multiplication.

(b) Prove that: (A∩B) U (B∩C) U (C∩A) = (AUB) ∩ (BUC) ∩ (CUA).

(c) If in a group (G, ○) the order of an element a is 5, then find the order of a^{18}.

5 + 4 + 3 = 12

Group – C

4. (a)

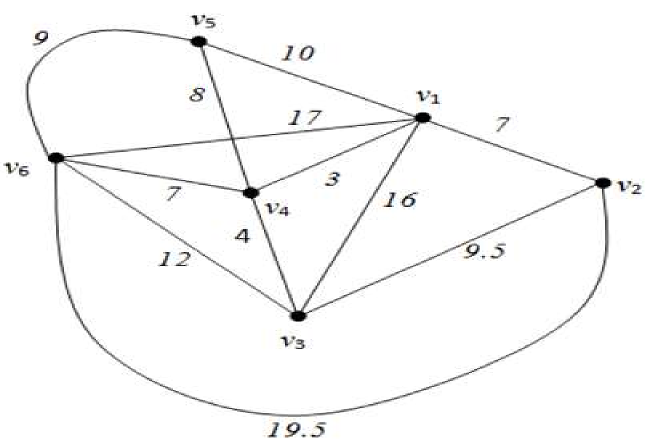

Fig. 1

Apply Prim's algorithm to find the minimum spanning tree for the above weighted connected graph in Fig. 1.

(b) Prove that in a graph G, there is an even number of vertices of odd degree.

8 + 4 = 12

5. (a) State the Konigsberg bridge problem. Prove that the sum of the degrees of all the vertices of a graph is equal to twice the number of vertices and from that prove that the maximum number of edges of a simple graph with n vertices is $n(n-1)/2$.

(b) Write short notes on the following :

(i) Bipartite graph

(ii) Planar graph

(iii) Minimum-Spanning tree.

(2 + 2 + 2) + (2 + 2 + 2) = 12

Group – D

6. (a) In how many ways the letters of the word "STRANGE" be arranged so that

(i) the vowels come together

(ii) the vowels never come together

(iii) the vowels occupy only odd places.

(b) Find the coefficient of X^{20} from the expansion $(X^3 + X^4 + \ldots\ldots\ldots)^3$.

(c) Prove that if any 30 people are selected, then we may choose a subset of 5 so that all 5 were born on the same day of the week.

4 + 4 + 4 = 12

7. (a) Solve the following recurrence relation by substitution:

$$a_n = a_{n-1} + 3^n, n \geq 1, a_0 = 1$$

(b) Find the recurrence relation for the sequence 1, 3, 5, 7, 15, 31,..........

(c) Find the solution to the recurrence relation: $a_n = a_{n-1} + a_{n-2}$ where, a_0=1 and a_1=1.

4 + 4 + 4 = 12

Group – E

8. (a) Prove that sum of all minterms of a boolean function for n number of variables = 1.

(b) Design a DFA that accepts even number of a's and b's.

6 + 6 = 12

9. (a) Simplify the following boolean function using K-Map:

$F = A'B'C' + B'CD' + A'BCD' + AB'C'$.