DISCRETE MATHEMATICS
(MCAP 1104)

Time Allotted : 3 hrs　　　　　　　　　　　　　　　**Full Marks : 70**

Figures out of the right margin indicate full marks.

***Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.***

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following:　　　**10 × 1 = 10**

    (i)　If the general term of the sequence $\{a_k\} = a^k$, then the corresponding generating function is
    (a) $\frac{1}{(1-ax)}$　　　(b) $\frac{1}{(1-x)}$　　　(c) $\frac{k}{(1-ax)}$　　　(d) $\frac{a}{(1-ax)}$

    (ii)　If n be the number of vertices, e be the number of edges and k be the number of components then
    (a) $e > n - k$　　　　　　　　(d) $e \geq n - k$
    (c) $e \leq n - k$　　　　　　　　(d) *none of these*.

    (iii)　If the function f : R→R is defined by f(x) = $\begin{cases} 3x - 4, & when \ x > 0 \\ -3x + 2 & when \ \ x \geq 0 \end{cases}$
    $then , f^{-1}(-2) =$
    (a) {2}　　　　　　　　　　(b) {−2,2}
    (c) {0,2}　　　　　　　　　(d) none of these.

    (iv)　Find the rank of the word LETTER, when the letters are arranged as in dictionary.
    (a) 13　　　　(b) 14　　　　(c) 15　　　　(d) 16.

    (v)　A Spanning-Tree of a connected graph G is a
    (a) graph containing all the vertices
    (b) graph containing all the edges
    (c) sub graph containing all the vertices and acyclic
    (d) none of the above.

    (vi)　Shortest path of a connected graph is obtained by applying
    (a) prim's algorithm　　　　　　(b) kruskal's algorithm.
    (c) dijkstra's algorithm　　　　(d) all the above.

    (vii)　The power set of an empty set has exactly _____ subset.
    (a) one　　　　(b) two　　　　(c) zero　　　　(d) three

    (viii)　If a set has 3 elements then A×A×A has
    (a) 9 elements　　　　　　　　(b) 27 elements
    (c) 6 elements　　　　　　　　(d) 19 elements.

    (ix)　If cardinality (A+B) = cardinality (A) + cardinality (B), then
    (a) A is a subset of B　　　　　(b) B is a subset of A
    (c) A and B are disjoint sets　　(d) none of these.

    (x)　Language of finite automata is
    (a) type-0　　　(b) type-1　　　(c) type-2　　　(d) type-3.

Group – B

2. (a)　Prove that $(P \wedge (P \leftrightarrow Q)) \to Q$ is a tautology.

    (b)　A relation ρ is defined on the set N of natural numbers such that "m ρ n iff m is a divisor of n" for all m, n ∈ N. Examine if ρ is
    (i) Reflexive　　(ii) Symmetric　　(iii) Transitive.
    3 + (3 + 3 + 3) = 12

3. (a)　Prove that for n variables the sum of all minterms = 1.

    (b)　Let A = {a, b} and P(A) be it's power set. Let ⊆ be the inclusion relation on the elements of (P(A), ⊆). Draw the hasse diagram for (P(A), ⊆).
    6 + 6 = 12

Group – C

4. (a)　Find the coefficient of x⁹ in $(1+x+x^2+...........+x^5)^4$.

    (b)　Solve the recurrence relation by generating function,
    $a_n - 7a_{n-1} + 10a_{n-2} = 0$ for n ≥ 2, where $a_0 = 10$, $a_1 = 41$.
    6 + 6 = 12

5. (a)　What is the number of permutations in the letters of the word MISSISSIPPI where, 4S's don't come together?

    (b)　Consider a set of integers from 1 to 250. Find how many of these numbers are divisible by 3 or 7 or 5. Also indicate how many are divisible by 3 or 7 but not by 5 and divisible by 3 or 5.

    (c)　Find the integer solution for the Diophantine equation 195x + 42y = 12.
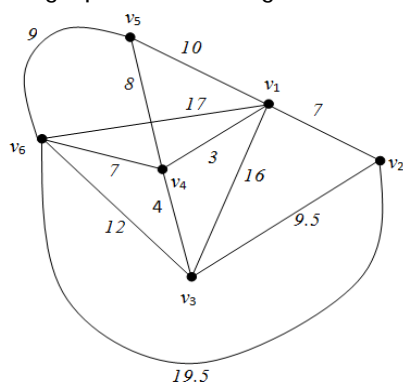   **3 + 4 + 5 = 12**

Group – D

6. (a) Prove that a tree with n vertices always has (n-1) edges.

   (b) Prove that the maximum number of edges in a graph G with n vertices and k components is $(n - k)(n - k + 1)/2$.

   6 + 6 = 12

7. (a) Apply Prim's algorithm to draw the minimum spanning tree for the above connected weighted graph shown in Fig.1.

Fig.1

   (b) Define the following terms:
   (i) Walk (ii) Path (iii) Bipartite graph (iv) Isomorphism of a graph.

   6 + (1.5 × 4) = 12

Group – E

8. (a) What is regular expression and regular language? Show that the language $\{0^n 1 0^n : n \geq 1\}$ is not a regular language. Define context sensitive language with example.

   (b) Consider the grammar G with V = {S, A,B}, Σ = {a, b} and
   P= { S-> aAS, S->a, A->SbA, A->ba }
   Construct the derivation tree of the word aabbaa.

   (2 + 4) + (2 + 4) = 12

9. (a) Construct a regular grammar for the language L={ 0^n 11| n≥1}.

   (b) Design a DFA that accepts even number of a's and b's.

   (c) What is the Mealy machine?

   4 + 6 + 2 = 12