

- (b) Find the real root of $x^3 - x - 11 = 0$ using bisection method.
- (c) Evaluate $y(0.2)$ using Runge-Kutta method of fourth order to calculate $\frac{dy}{dx} = y + x$, $y(0) = 1$ taking $h = 0.1$.

5 + 3 + 4 = 12

9. (a) Evaluate $y(0.02)$ using Euler's modified method to calculate $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ taking $h = 0.01$.

- (b) Solve the following system of equations using the Gauss Seidel method.
 $4x + 3y - 5z = 2$, $x + y - z = 1$, $8x + y - 5z = 4$.

- (c) Find $\sqrt{45}$ using Newton-Raphson method.

4 + 5 + 3 = 12

**NUMERICAL AND STATISTICAL TECHNIQUES
(MCAP 1103)**

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

**Group – A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) The chance that a leap year selected at random will contain 53 Wednesdays is
 (a) $\frac{2}{7}$ (b) 0 (c) 1 (d) $\frac{6}{7}$
- (ii) X and Y are both negative random variables, mutually independent, $E(XY) = 6$ and $|E(X)| = 2$. Then the value of $E(Y)$ is
 (a) 3 (b) -3 (c) 12 (d) ± 3
- (iii) If X is normally distributed with mean 0 and variance 1, then the expectation of X^2 is
 (a) 1 (b) 2 (c) 8 (d) 0
- (iv) If T_1 and T_2 are statistics with $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, then the unbiased estimator of θ_1 is
 (a) $3T_2 - T_1$ (b) $3T_2 + 2T_1$
 (c) $2T_2 - 3T_1$ (d) $3T_2T_1$
- (v) Which of the following is Type II error?
 (a) The error of accepting H_0 when H_0 is false
 (b) The error of accepting H_0 when H_0 is true
 (c) The error of rejecting H_0 when H_0 is true
 (d) The error of rejecting H_0 when H_0 is false.

- (vi) If $f(x)$ is continuous in the interval (a, b) and if $f(a)$ and $f(b)$ are opposite signs, then there is
- at least one root of $f(x) = 0$ between a and b
 - at most one root of $f(x) = 0$ between a and b
 - there is no real root of $f(x) = 0$ between a and b
 - none of these.
- (vii) Which of the following does not always guarantee the convergence?
- Secant method
 - Regular falsi method
 - Bisection method
 - Newton Raphson method.
- (viii) The degree of precision of Simpson's 1/3 rule is
- 0
 - 2
 - 1
 - 3.
- (ix) The number significant figure in 0.0340980 is
- five
 - six
 - seven
 - three.
- (x) The relation between shift operator 'E' and forward difference operator 'Δ' is given by
- $\Delta = 1 - E$
 - $E = 1 + \Delta$
 - $E = \Delta$
 - $E = \Delta + 2.$

Group – B

2. (a) For any two events A_1 and A_2 , prove that
- $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$
 - $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$
 - $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$
- (b) Find the probability that in the throw of two unbiased dice the sum of points will be even or less than 5.
- (5 + 1 + 1) + 5 = 12**
3. (a) Three balls are drawn at random from a bag containing 5 white and 3 black balls. If X denotes the number of white balls drawn, find the probability distribution of X , $E(X)$ and $\text{Var}(X)$.
- (b) State Chebyshev's inequality and explain its significance. The length of a knife is on average 5 inches long, with a standard deviation of 1/10th of 1 inch. What % of observations will be between 4.75 and 5.25 inches long?
- (3 + 1 + 2) + (2 + 1 + 3) = 12**

Group – C

4. (a) If X_1, X_2, X_3, X_4, X_5 and X_6 are independent simple random sample from a normal population with unknown variance σ^2 , find k so that $k [(X_1 - X_2)^2 + (X_3 - X_4)^2 + (X_5 - X_6)^2]$ is an unbiased estimator of σ^2 .
- (b) A random sample of 200 apples was drawn from a very large-sized consignment and 25 of them were found to be bad. Find the unbiased estimate of variance of sample proportion of bad apples.
- 6 + 6 = 12**
5. (a) Define the following, in context of hypothesis testing: power of a test, level of significance and unbiased test.
- (b) In order to test whether a coin is unbiased, the coin is tossed 5 times and the null hypothesis of unbiasedness of the coin is rejected if no head or 5 heads are obtained. (i) What is the probability of Type I error? (ii) Find the probability of Type II error when the corresponding probability of head is 0.3.
- (2 + 2 + 2) + (3 + 3) = 12**

Group – D

6. (a) What is relative error and percentage error?
- (b) Given: Construct the difference table and compute i) $f(1.5)$ ii) $f(7.5)$. From the table given below.
- | | | | | | | | | |
|---|---|---|----|----|-----|-----|-----|-----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| y | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 |
- 2 + 10 = 12**
7. (a) Establish Newton's forward interpolation formula.
- (b) Evaluate $\int_0^1 \sqrt{1-x^3} dx$ using Simpson's 1/3rd rule dividing the range into 10 intervals.
- (c) What is the geometrical significance of Trapezoidal method?
- 5 + 4 + 3 = 12**

Group – E

8. (a) Evaluate $y(0.1)$ using Euler's method $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.02$.