(ii) Find out the normal mode frequencies and normal coordinates.
(iii) Express the total energy of the system in terms of normal co-ordinates and velocities.

[4 + (3 + 3) + 2] = 12

Group – E

8. (a) Define the bulk modulus. For a homogeneous and isotropic material, derive the expression for the bulk modulus in terms of its Young's modulus and Poisson's ratio.

(b) A metallic block in the shape of a cube of side 50 cm is fixed firmly on a flat table. On the top side of the cube, a shearing force of 10000 N is applied. The shear angle is found to be 0.0006°. Calculate the shear modulus for the metal.

(c) Consider an arbitrarily shaped body in equilibrium under the action of several external forces. Draw a suitable elementary rectangular parallelepiped within the body. Show the normal and shear stress components acting on each face of this parallelepiped. You may assume the components to be positive

(d) Now consider a thin sheet of a metal which is being pulled by forces in the plane of the sheet under which the sheet is in equilibrium, so that plane stress conditions are satisfied. Using a suitable parallelepiped, develop the relation between the normal and shear stresses at each point within this metal under these conditions.

(1 + 3) + 2 + 3 + 3 = 12

9. (a) What is the difference between the Lagrangian and Eulerian descriptions of motion? Which description is, in general, used to study the mechanics of fluids and why?

(b) Develop the equation of continuity in differential form. Consider an incompressible fluid in two-dimensional flow in the xy plane. If the x-component of the velocity field, $\vec{V}(t)$, is $u = 3\sinh x$, find the expression for $\vec{V}(t)$ if its y-component at $y = 0$ is $v = \cosh x$.

(c) Verify that $\frac{D\vec{V}}{Dt} = (\vec{V}.\vec{\nabla})\vec{V}$ assuming a steady flow.

(1 + 2) + (4 + 2) + 3 = 12

PHYSICS - II
(PHYS 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A system of particles consists of a 3 gm mass located at (1, 0, -1), a 5 gm mass at (-2, 1, 3) and a 2 gm mass at (3, -1, 1). Find the 'y' coordinate of the centre of mass.
(a) 0.3 (b) -0.1 (c) 1.4 (d) 3.

(ii) The number of degrees of freedom for a rigid body which has one point fixed but can move in space about this fixed point are
(a) 5 (b) 3 (c) 4 (d) 2.

(iii) The cyclic coordinate in the Lagrangian $L = \frac{1}{2}(\dot{q_1}^2 + q_1\dot{q_2}^2 + \dot{q_3}^2) - V(q_1, q_2)$ are
(a) q_1, q_2 (b) q_1, q_3 (c) q_2, q_3 (d) q_3

(iv) A system has three particles and one holonomic constraint. The dimension of its configuration space is
(a) 3 (b) 6 (c) 7 (d) 8.

(v) The relation between the action A and lagrangian L is given by
(a) $A = \int Ldt$ (b) $L = \int Adt$ (c) $A = \int tdL$ (d) $L = \int tdA$

(vi) Hamilton's equation of motion determines the trajectory in
(a) configuration space (b) dual space
(c) phase space (d) Krein space.

(vii) If V_{jk} represents the potential energy matrix for system performing small oscillation then
(a) $V_{jk} = V_{kj}$ (b) $V_{jk} = -V_{kj}$ (c) $V_{jk} = V_{jk}^2$ (d) $V_{jk} = -V_{jk}^2$

(viii) Consider a material in the form of a cube in uniform shear. This is equivalent to the following forces on the cube:
(a) Compressive forces on top and bottom and equal stretching forces on two sides
(b) Compressive forces on top and bottom and equal compressive forces on two sides
(c) Stretching forces on top and bottom and equal stretching forces on two sides
(d) Equal compressive forces on two sides.

(ix) The Navier-Stokes equation is
(a) the continuity equation for a fluid
(b) the momentum equation for an inviscid fluid
(c) the energy equation for a viscous fluid
(d) none of the above.

(x) Find the correct relation between Young's modulus and Poisson's ration
(a) $Y = 2\mu(1+\nu)$ (b) $Y = \frac{\mu}{2(1+\nu)}$ (c) $Y = \frac{\nu}{2(1+\mu)}$ (d) $Y = 3\mu(1+\nu)$
Where, Y $\rightarrow$ Young's modulus, $\nu \rightarrow$ Poisson's ratio, $\mu \rightarrow$ Modulus of rigidity.

Group – B

2. (a) A uniform plate has the shape of the region bounded by the parabola $y = x^2$ and $y = H$ in the xy plane. Find the centre of mass.

(b) If a rigid body with one point fixed rotates with angular velocity ω and has angular momentum L, prove that the kinetic energy is given by $T = \frac{1}{2}\vec{\omega}.\vec{L}$
Also prove that kinetic energy can be written as
$T = \frac{1}{2}(I_{xx}w_x^2 + I_{yy}w_y^2 + I_{zz}w_z^2 + 2I_{xy}w_xw_y + 2I_{xz}w_xw_z + 2I_{yz}w_yw_z)$,
symbols have their usual meaning.

(c) Write down the relation between moments of inertia about two parallel set of axes (x, y, z) and (x', y', z') respectively, such that one set of axes passes through the centre of mass (at point O) of the rigid body and the origin of the other set is shifted to point O' where $OO' = \vec{a}$.
Using this relation, find the moment of inertia about O', where
$I = \begin{bmatrix} \frac{1}{4}MR^2 & 0 & 0 \\ 0 & \frac{1}{4}MR^2 & 0 \\ 0 & 0 & \frac{1}{2}MR^2 \end{bmatrix}$ and $\vec{a} = 2\hat{j}$.

4 + (2 + 2) + (2 + 2) = 12

3. (a) Three equal point masses m are located at (a, 0, 0), (0, a, 2a) and (0, 2a, a). Show that the moment of inertia tensor is not a diagonal matrix. Find the principal moment of inertia about the origin and a set of principal axes.

(b) Write down the Euler's equations of motion of a rigid body. A rigid body which is symmetric about an axis has one point fixed on this axis. Discuss the rotational motion of the body, assuming that there are no forces acting other than the reaction force at the fixed point.

(2 + 2 + 2) + (2 + 4) = 12

Group – C

4. (a) What do you mean by the Brachistrochrone problem? Show that the Brachistrochrone under constant gravity is a cycloid.

(b) The Lagrangian of a system is given by $L = \frac{1}{2}(\dot{q}_1{}^2 + \dot{q}_2{}^2 + q_1\dot{q}_2 - q_2\dot{q}_1) - \frac{1}{2}(q_1{}^2 + q_2{}^2)$, where symbols have their usual meaning.
(i) Find out the components of generalized momentum.
(ii) Find out the Lagrange equations of motion.

(2 + 4) + (2 + 4) = 12

5. (a) Show that if Hamiltonian of a system is not an explicit function of time then it is a conserved quantity.

(b) A simple pendulum of mass m and string length l is hanging from a fixed point of suspension. (i) Write down the Cartesian coordinates of the system in terms of a suitable generalized coordinates. (ii) Construct the Lagrangian of the system. (iii) Hence construct the Hamiltonian of the system.

3 + (2 + 4 + 3) = 12

Group – D

6. (a) A particle of mass m has an equation of motion $\ddot{x} + \omega_0^2 x = 0$. Show that total energy is conserved for the system.

(b) Three identical springs of spring constant k and two blocks of masses m and $2m$ are used to create a coupled system such that the two masses are connected between the three springs and the motion is linear. There is no friction. (i). Construct the Lagrangian of the system. (ii) Find out the normal frequencies and normal coordinates.

3 + (3 + 6) = 12

7. The kinetic (T) and potential (V) energies of a two-particle coupled spring-mass system is given by
$T = \frac{1}{2}\dot{q}_1{}^2 + 2\dot{q}_2{}^2$ and $V = \frac{1}{2}(5q_1{}^2 + 20q_2{}^2 - 6q_1q_2)$.
Symbols have their usual meaning.
(i) Construct the matrices V_{jk} and T_{jk}.