SIGNALS & SYSTEMS
(ELEC 3103)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The function $f(t) = \frac{\sin(\pi t)}{\pi t}$, for $-\infty < t < \infty$, oscillates with increasing t. The nature of function is

(a) even function (b) odd function
(c) odd periodic function (d) even periodic function.

(ii) The even component of the signal $x(t) = u(t) - u(t-1)$ with $u(t) = unit\ step\ function$ is

(a) $x_e(t) = \begin{cases} 1 & for\ -1 \le t \le 2 \\ 0 & ;\ elesewhere \end{cases}$

(b) $x_e(t) = \begin{cases} 0.5 & for\ -1 \le t \le 1 \\ 0 & ;\ elesewhere \end{cases}$

(c) $x_e(t) = \begin{cases} 0.5 & for\ 0 \le t \le 1 \\ -0.5 & ;\ elesewhere \end{cases}$

(d) $x_e(t) = \begin{cases} 0.5 & for\ 0 \le t \le 1 \\ -0.50 & ;\ -1 \le t \le 0 \end{cases}$

(iii) The Parseval's identity states that the energy (E) content of the signal $x(t)$ is equal to

(a) $E = \int_{-\infty}^{\infty} \left|\frac{x(t)}{\sqrt{2}}\right|^2 dt = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

(b) $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left|\frac{1}{2\pi} X(\omega)\right|^2 d\omega$

(c) $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$

(d) $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 2\pi \int_{-\infty}^{\infty} \left|\frac{X(\omega)}{\sqrt{2}}\right|^2 d\omega$

(iv) The R.O.C of $z-transform$ for the discrete signal $x(n) = 2(3^n)u(-n)$ (non-causal signal) is
(a) R.O.C : Outside the unit circle of $z-$ plane
(b) R.O.C : $|z| > 3$
(c) R.O.C : Complete $z-$ complex plane
(d) R.O.C : $|z| < 3$.

(v) A Fourier Transform $X(\omega)$ of a real aperiodic signal $x(t)$ will have the amplitude and phase spectrum characteristics as
(a) even and even spectrums
(b) odd and even spectrums
(c) even and odd spectrums
(d) odd and oven spectrums.

(vi) The transfer function $H(z)$ of the system represented by the given difference equation $y(n) = y(n-1) + y(n-2) + x(n) + 2x(n-1)$ with zero initial conditions, is
(a) $H(z) = \frac{1+2z^{-1}}{1+z^{-1}+z^{-2}}$ (b) $H(z) = \frac{1+z^{-1}}{1+z^{-1}+z^{-2}}$
(c) $H(z) = \frac{1+z^{-1}}{1-z^{-1}-z^{-2}}$ (d) $H(z) = \frac{1+2z^{-1}}{1-z^{-1}-z^{-2}}$.

(vii) The convolution integral of two signals $(i.e.\ h(t) * x(t))$ $h(t) = e^{-2t}u(t)$ and $x(t) = e^{-3t}u(t)$ is
(a) $(e^{-3t} - e^{-2t})u(t)$ (b) $(e^{-2t} + e^{-3t})u(t)$
(c) $(e^{-2t} - e^{-3t})u(t)$ (d) $(e^{-2t})(e^{-3t})u(t)$.

(viii) A unit step response for a standard form of a first-order system $G(s) = \frac{K}{s\tau+1}$ takes 2 second to reach 63.2% of its final value 10 units. The transfer function of the system $G(s)$ is
(a) $G(s) = \frac{2}{2s+1}$ (b) $G(s) = \frac{10}{2s+1}$
(c) $G(s) = \frac{10}{s+\frac{1}{2}}$ (d) $G(s) = \frac{1}{2s+1}$.

(ix) The transfer function $G(s)$ for the state-space system matrices $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $c = [1 \quad 0]$ is given by
(a) $G(s) = \frac{s}{s^2-1}$ (b) $G(s) = \frac{s}{s^2+1}$
(c) $G(s) = \frac{1}{s^2+1}$ (d) $G(s) = \frac{1}{s^2-1}$.

9. (a) A series $R-L-C$ circuit having the circuit parameters $R = 2\Omega$, $L = 1H$ and $C = 0.2F$ respectively, is excited with an input $u(t)$. Obtain the state space model of the system, assuming the voltage across the capacitor $(x_1(t) = v_c(t))$ and its derivative $(x_2(t) = \dot{v}_c(t))$ as the state variables.

(b) Convert the state-space model as obtained in Q.9 (a) to an equivalent transferfunction model $G(s)$. Is the system stable? Justify your answer.

6 + 6=12

(x) The steady-state output $(y(t))$ response of a first order system $G(s) = \frac{3}{(s+0.4)}$ due to sinusoidal input $u(t) = sin2t$ is
(a) $y(t) = 3\sin(2t + 158.6(deg.))$
(b) $y(t) = 1.47\sin(2t + 1.38(rad))$
(c) $y(t) = 3\sin(2t - 158.6(deg.))$
(d) $y(t) = 1.47\sin(2t - 2.77(rad))$.

Group – B

2. (a) Prove that $\delta(at) = \frac{1}{|a|}\delta(t)$, where, $\delta(t)$ is unit impulse signal and 'a' is a real number.

(b) Find out the normalized energy and power of the signal $x(t) = e^{-2t}u(t)$ and hence comment on whether it is an energy or a power signal.

(c) Find the graphical convolution of the following two signals $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$.

3+ 3 + 6 =12

3. (a) Find the Exponential Fourier Series for the signal shown in Fig.1. Hence, also find out the trigonometric Fourier Series coefficients.

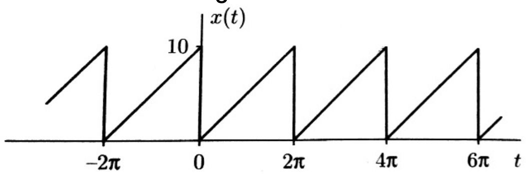

Fig.1

(b) Obtain the Fourier Transform of the signal $x(t)$ shown in Fig.2 and also sketch its amplitude spectrum.

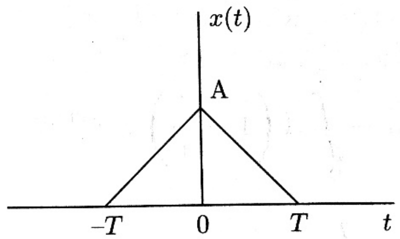

Fig.2

(4 + 2) + (5 + 1) =12

Group – C

4. (a) If $x(n) \leftrightarrow X(z)$ with R.O.C R_x then show that $\lim_{n\to\infty} x(n) = \lim_{n\to 1}(1 - z^{-1})$ provided $x(\infty)$ exists.

(b) Find the impulse response of the causal discrete-time system with transfer function $G(z) = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$ using inverse integral (residue) method. Discuss the pitfalls of this method.

5 +7 = 12

5. (a) If $x_1(n) \leftrightarrow X_1(z)$; R.O.C: R_{x_1} and $x_2(n) \leftrightarrow X_2(z)$; R.O.C: R_{x_2} then show that z-transform of $[x_1(m) * x_2(m)] \leftrightarrow X_1(z)X_2(z)$.

(b) Obtain the input-output expression in difference equation from the causal transfer function model $G(z) = \frac{0.25z^{-1}+0.2z^{-2}}{1-0..6z^{-1}+0.05z^{-2}}$. Is the given system stable? Find the R.O.C of the given transfer function $G(z)$.

5+7=12

Group – D

6. (a) Show that a causal LTI continuous system is input-output stable if, and only if, its impulse response $h(t)$ satisfies $\int_0^\infty |h(t)|dt < \infty$.

(b) For the impulse response $h(t) = (1 - e^{-4t})\,u(t)$ of a system, determine whether the corresponding system is (i) causal (ii) stable (based on the above definition of stability).

6 + 6 = 12

7.(a) State and explain Lagrange's equation for the analysis of the motion of a dynamic system.

(b) What do you mean by the frequency response of a continuous time system $G(s)$? Discuss briefly.

5+7 = 12

Group – E

8. (a) Define the following terms: State, State vector and State space of a dynamical system.

(b) Consider the state space model given below:

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -4\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1\\ 2\end{bmatrix}u(t)\ (state\ eq.);$$

$$y(t) = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\ (output\ eq.)$$

Obtain the solution of state space equation and output response with the following initial conditions $\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}1\\ 2\end{bmatrix}$ and unit impulse input $(t) = \delta(t)$.

5 + 7 = 12