

Now, answer the following questions:

- i) Give a Type 2/ context-free grammar that generates the set  $L_7$   
 ii) Convert the grammar that you design in part (i) to its equivalent push down automata (PDA).

- (b) Explain acceptance by empty stack and acceptance by final state for a PDA.  
**(5 + 5) + 2 = 12**

7. (a) Let  $L_1 = L(S)$  and  $L_2 = L(T)$ , where the non-terminal symbols  $S$  and  $T$  satisfy the productions:  $S \rightarrow a \mid b \mid abSS$ ,  $T \rightarrow a \mid b \mid TTab$ . Find examples of strings  $w$  of length twenty such that:

- (i)  $w \in L_1$  and  $w \in L_2$ . (ii)  $w \in L_1$  and  $w \notin L_2$ .  
 (iii)  $w \notin L_1$  and  $w \in L_2$ . (iv)  $w \notin L_1$  and  $w \notin L_2$ .

- (b) Consider the unrestricted grammar over the singleton alphabet  $\Sigma = \{a\}$ , having the start symbol  $S$ , and with the following productions.  $S \rightarrow AS \mid aT$ ,  $Aa \rightarrow aaaA$ ,  $AT \rightarrow T$ ,  $T \rightarrow \epsilon$ . What is the language generated by this unrestricted grammar? Justify.

- (c) Identify and remove the non-reachable and non-generating nonterminals from the following grammar:

$S \rightarrow XY1 \mid 0$      $X \rightarrow 00X \mid 1$      $Y \rightarrow 1X1$      $Z \rightarrow 00$   
**(4 × 1.5) + 3 + 3 = 12**

### Group – E

8. (a) Design a Turing machine  $M$  over  $\{a, b\}$  such that  $L(M) = \{x \mid \text{length of } x \text{ is odd}\}$ . Explain the working of this machine taking two suitable examples, where one of them will be accepted and another will be rejected.

- (b) Prove that the halting problem in Turing Machines is undecidable.  
**(5 + 3) + 4 = 12**

9. (a) Design a Turing machine  $M_2$  that decides  $L_9 = \{0^{2^n} \mid n \geq 0\}$ .

- (b) Let  $L_{10}$  be a context-free language. Is  $L_{10}$  necessarily a recursive set? Answer YES or NO. Give reasons for your answer.  
**7 + (1 + 4) = 12**

## FORMAL LANGUAGE & AUTOMATA THEORY (CSEN 3101)

Time Allotted : 3 hrs

Full Marks : 70

*Figures out of the right margin indicate full marks.*

**Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.**

**Candidates are required to give answer in their own words as far as practicable.**

### Group – A (Multiple Choice Type Questions)

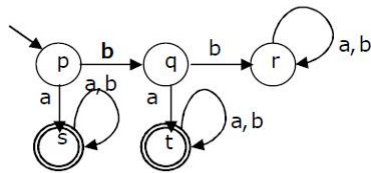
1. Choose the correct alternative for the following: **10 × 1 = 10**

- (i) Context free languages are not closed under  
 (a) Union (b) Concatenation (c) Intersection (d) None of these.
- (ii) Consider the grammar  $G = (\{S\}, \{a\}, \{S \rightarrow SS\}, S)$ . Which of the following is true about  $L(G)$ ?  
 (a)  $L(G) = \{a\}$  (b)  $L(G) = \{aa\}$  (c)  $L(G) = \Phi$  (d) All of these.
- (iii) Suppose we have a parse tree of a CNF grammar  $G = (V_N, \Sigma, P, S)$ , and also assume that the yield of the tree is a terminal string  $w$ . If the length of the longest path is  $\leq n$ , then which of the following is true?  
 (a)  $|w| \leq 2^{n-1}$  (b)  $|w| < 2^{n-1}$  (c)  $|w| = 2^{n-1}$  (d) None of these.
- (iv) Consider the following identities for regular expressions:  
 (i)  $(r + s)^* = (s + r)^*$  (ii)  $(r^*)^* = r^*$  (iii)  $(r^* s^*)^* = (r + s)^*$   
 Which of the above identities are true?  
 (a) (i) and (ii) only (b) (ii) and (iii) only  
 (c) (iii) and (i) only (d) (i), (ii) and (iii).
- (v) Given the language  $L = \{ab, aa, baa\}$ , which of the following strings are in  $L^*$ ?  
 (1) abaabaabaa (2) aaaabaaaa (3) baaaaabaaaab (4) baaaaabaa  
 (a) 1, 2 and 3 (b) 2, 3 and 4  
 (c) 1, 2 and 4 (d) 1, 3 and 4.
- (vi) Recursively enumerable languages are not closed under:  
 (a) Concatenation (b) Complement (c) Union (d) Intersection.

- (vii) Which one of the following languages over the alphabet  $\{0,1\}$  is described by the regular expression:  $(0+1)^*0(0+1)^*0(0+1)^*$ ?
- The set of all strings containing the substring 00
  - The set of all strings containing at most two 0's
  - The set of all strings containing at least two 0's
  - The set of all strings that begin and end with either 0 or 1.
- (viii) If every production of a CFG  $G = (V_N, \Sigma, P, S)$  is of the form  $A \rightarrow \alpha\alpha$ , where  $A \in V_N$ ,  $\alpha \in \Sigma^*$ , then which of the following normal form the given grammar satisfies?
- GNF
  - CNF
  - both (a) and (b)
  - none of these.
- (ix) Which of the following problems is undecidable?
- Deciding if a given CFG is ambiguous
  - Deciding if a given string is generated by a given CFG
  - Deciding if a language generated by a given CFG is empty
  - Deciding if a language generated by a given CFG is finite.
- (x) Let 'X' be set of all languages accepted by deterministic push down automata (DPDA) by final state and 'Y' be set of all languages accepted by DPDA by empty stack then, which of the following is true?
- X is proper subset of Y
  - X = Y
  - X is proper super set of Y
  - none of the above.

**Group – B**

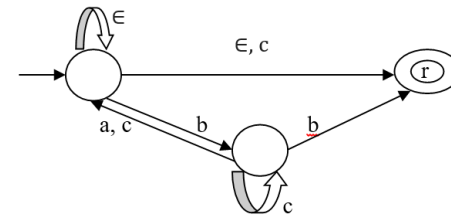
2. (a) Construct an NFA to accept the regular expression  $b(((ba)^* + bbb)^* + a)^*b$ , such that the number of states are as minimum as possible.
- (b) A deterministic finite automation (DFA)D with alphabet  $\{a,b\}$  is given below. Find the minimized DFA from it.



- (c) Let  $\Sigma = \{0, 1\}$ . Give DFA accepting the set of all strings, when interpreted in reverse as a binary integer, is divisible by 3.
3. (a) "For a Mealy machine, if the input string is of length n, then the output string is of length n+1; but in case of Moore machine, if the input string is of length n, the output string is also of the same length n". State whether the given statement is True or False. Justify your answer with suitable example.

**3 + 4 + 5 = 12**

- (b) Define epsilon closure of a state in nondeterministic finite automata (NFA). Consider the following epsilon NFA on  $\Sigma = \{a, b, c\}$ , where p is the initial state, r is the final state and  $\epsilon$  represents null symbol:



Find the equivalent DFA of the given epsilon NFA.

**(1 + 2) + (2 + 7) = 12**

**Group – C**

4. (a) Let  $L_1$  be the following language defined on the input alphabet  $\Sigma = \{0, 1\}$ .  $L_1 = \{ \alpha \mid \text{the string } \alpha \text{ does not contain the substring '00'} \}$ . Thus the strings 101101 and 01110 are both in  $L_1$ , but the string 1001 is not in  $L_1$ . Now, answer the following questions:
- Give a regular expression for  $L_1$ .
  - Design a NFA from the regular expression that you obtained at part (i)

- (b) Consider the following language  $L_2$ :

$$L_2 = \{ 1^n \mid n \geq 0 \}$$

Prove that  $L_2$  is not a regular language by using pumping lemma.

**(3 + 5) + 4 = 12**

5. (a) Write down the Pumping Lemma for regular languages. Use the Pumping Lemma to prove that the following language is non-regular.  $L = \{b^p ab^q \mid p, q \geq 0, |p - 2q| = r^2, r > 0\}$ .
- (b) Write regular expressions for the following regular sets:
- Set of all strings over  $\{a, b\}$  containing exactly two a's and two b's
  - Set of all strings over  $\{0, 1\}$  containing at most one 0
  - Set of all strings over  $\{0, 1\}$  containing no two consecutive 0's and 1's
  - Set of all strings over  $\{0, 1\}$  containing even number of characters.

**(2 + 4) + (1.5 × 4) = 12**

**Group – D**

6. (a) The language  $L_7$  is defined on the alphabet  $\Sigma = \{ '(', ')', '{', '}' \}$  and consists of all sequences of well-formed (i.e., balanced) parentheses and braces. Thus the sequences  $'(())'$ ,  $'(())()'$ ,  $'\{\}$ ',  $'(())'$  are in  $L_7$ , but the sequences  $'()()'$ ,  $'\{\}'$  are not in  $L_7$ .