(b) Prove that $\int_0^{\frac{\pi}{2}}\int_0^{\pi}\sin(x+y)dxdy = 2$.

6 + 6 = 12

9. (a) Find the directional derivative of the scalar point function $f(x,y,z) = x^2 + xy + z^2$ at the point $A = (1,-1,-1)$ in the direction of $\vec{AB}$ where, B has coordinates $(3,2,1)$.

(b) A vector field $\vec{F}$ is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral $\int_\Gamma \vec{F}.d\vec{r}$, where, Γ is the circular path given by $x^2 + y^2 = a^2$.

6 + 6 = 12

MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The matrix $\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$ is

(a) symmetric (b) skew-symmetric
(c) singular (d) orthogonal.

(ii) The two eigenvalues of the matrix $A = \begin{bmatrix}2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{bmatrix}$ are 2 and -2. The third eigenvalue is

(a) 1 (b) 0 (c) 3 (d) 2.

(iii) If Rolle's theorem is applicable to $f(x) = x(x^2-1)$ in $[0,1]$, then $c =$

(a) 1 (b) 0 (c) $-\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{3}}$.

(iv) In the MVT $f(h) = f(0) + hf'(\theta h); 0 < \theta < 1$, if $f(x) = \frac{1}{1+x}$ and $h = 3$, then the value of θ is

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$.

(v) $f(x,y) = \frac{\sqrt{y}+\sqrt{x}}{y+x}$ is a homogeneous function of degree

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) 2.

(vi) $\int_0^{\frac{\pi}{2}} \cos^6 x dx =$

(a) $\frac{7\pi}{12}$ (b) $\frac{5\pi}{32}$ (c) $\frac{\pi}{32}$ (d) $\frac{3\pi}{16}$.

(vii) If $u(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(a) 0 (b) $2u(x,y)$ (c) $u(x,y)$ (d) 1.

(viii) If $y = e^{-x}$, then y_n is

(a) e^{-x} (b) $(-1)^n$ (c) $(-1)^n e^{-x}$ (d) 0.

(ix) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

(a) $p \geq 1$ (b) $p \leq 1$ (c) $p > 1$ (d) $p < 1$.

(x) The value of $\int_C (xdx - dy)$ where C is a line joining $(0,1)$ to $(1,0)$ is

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$.

Group – B

2. (a) Without expanding prove that

$$\begin{vmatrix} 1 & \alpha & \alpha^2 - \beta\gamma \\ 1 & \beta & \beta^2 - \gamma\alpha \\ 1 & \gamma & \gamma^2 - \alpha\beta \end{vmatrix} = 0$$

(b) Solve by Cramer's rule:

$$x + y + z = 1$$
$$ax + by + cz = k$$
$$a^2x + b^2y + c^2z = k^2$$

6 + 6 = 12

3. (a) Investigate for which values of λ and μ the following system of equations:

$$x + 2y + 3z = 6$$
$$x + 3y + 5z = 9$$
$$2x + 5y + \lambda z = \mu$$

has (i) no solution, (ii) unique solution, (iii) infinite number of solutions.

B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/1ST SEM/MATH 1101 (BACKLOG)/2019

(b) If $A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 3 & -1 \end{pmatrix}$ then verify that A satisfies its own characteristic equation. Hence find A^{-1}.

6 + 6 = 12

Group – C

4. (a) Verify Rolle's theorem for the following function: $f(x) = \sin x$ in $[0, \pi]$.

(b) Use mean value theorem to prove the following inequality: $0 < \frac{1}{x}\log_e \frac{e^x - 1}{x} < 1$.

6 + 6 = 12

5. (a) Show that the series $\frac{1}{1^2+1} + \frac{1}{2^2+1} + \frac{1}{3^2+1} + \ldots$ to ∞ is convergent.

(b) Test the convergence of the series $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots$ to ∞.

6 + 6 = 12

Group – D

6. (a) If $z = e^{ax+by} f(ax - by)$, show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

(b) Verify Euler's theorem for the function $f(x,y) = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$.

6 + 6 = 12

7. (a) If $u = f(x - y, y - z, z - x)$, then prove that $u_x + u_y + u_z = 0$

(b) If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$.

6 + 6 = 12

Group – E

8. (a) Evaluate $\int_C \{(2y+3)dx + xzdy + (yz - x)dz\}$, where, C is the arc of the curve $x = 2t^2, y = t, z = t^3$ from the point $(0,0,0)$ to the point $(2,1,1)$.