PROBABILITY AND STATISTICAL METHODS
(MATH 2111)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively. Then $P(X=1)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{32}$.

(ii) If μ is a parameter and $H_0 : \mu = 5$ is the null hypothesis, then which one of the following is left sided alternative hypothesis

(a) $H_1 : \mu \neq 5$ (b) $H_1 : \mu < 5$ (c) $H_1 : \mu > 5$ (d) $H_1 : \mu = 4$.

(iii) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then the interval in which k lies is

(a) $0 \le k \le \frac{1}{4}$ (b) $0 \le k \le 4$ (c) $-\frac{1}{4} \le k \le 0$ (d) $-4 \le k \le 0$.

(iv) The steady state probability vector for the probability transition matrix

$P = \begin{pmatrix} 0.5 & 0.5 \\ 1 & 0 \end{pmatrix}$ is

(a) $\left(\frac{2}{3}, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (c) $(1,0)$ (d) $(0,1)$.

(v) If S be the standard deviation of size 9 drawn from a normal population with standard deviation 2, then $\frac{9}{4}S^2$ has χ^2 distribution with degree of freedom

(a) 9 (b) 8 (c) 7 (d) 10.

(vi) A random variable X has Poisson distribution such that $P(1)=P(2)$, then $P(0)=$

(a) $\frac{1}{e}$ (b) $\frac{1}{e^2}$ (c) $\frac{1}{e^3}$ (d) $\frac{1}{e^4}$.

(vii) The joint probability density function of the random variable X and Y is given by $f(x,y) = \begin{cases} ce^{-(x+y)}, & 0 < y < x < \infty \\ 0, & otherwise \end{cases}$

Then the value of c is

(a) 1 (b) 2 (c) 5 (d) 7.

(viii) The smallest value of k in the Chebyshev's inequality $P(|X-\mu| < k\sigma) \ge 1 - \frac{1}{k^2}$ for which the probability is at least 0.99 is

(a) 0.01 (b) $\sqrt{0.01}$ (c) $\sqrt{10}$ (d) 10.

(ix) If $E(T_1) = \theta_1 + \theta_2$ and $E(T_2) = \theta_1 - \theta_2$ then the unbiased estimator of θ_1 is

(a) $T_1 + T_2$ (b) $\frac{1}{2}(T_1 - T_2)$ (c) $\frac{1}{2}(T_1 + T_2)$ (d) $T_1 - T_2$.

(x) The state 2 of the Markov Chain with transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix} \end{array}$$ is

(a) null recurrent (b) transient
(c) periodic (d) ergodic.

Group – B

2. (a) In a certain factory manufacturing razor blades, there is a small chance of $1/500$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective and (ii) two defective blades respectively in one consignment of 10,000 packets.

(b) Find the characteristic function of the Laplace distribution defined by the probability density function $f(x)$ given by

9. (a) The population of scores of children in a test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean of 16.9, find 95% confidence limits for the mean score of the population assuming that the population is normal.

(b) Let p denotes the probability of getting a head when a given coin is tossed once. Suppose that the hypothesis $H_0 : p = 0.5$ is rejected in favour of $H_1 : p=0.6$ if 10 trials result in 7 or more heads. Calculate the probabilities of Type-I and Type-II error. (The given table may be used).

6 + 6 = 12

$$f(x) = \frac{1}{2\lambda} e^{-\frac{|x-\mu|}{\lambda}}, -\infty < x < \infty, (\lambda > 0)$$

6 + 6 = 12

3. (a) A fair coin is tossed 400 times. Using normal approximation to binomial distribution find the probability of obtaining (i) exactly 200 heads, (ii) between 190 and 210 heads, both inclusive. Given that the area under standard normal curve between z = 0 and z = 0.05 is 0.0199 and between z=0 and z=1.05 is 0.3531.

(b) Find the probability density function of the continuous distribution which has the characteristic function $e^{-|t|}$.

6 + 6 = 12

Group – C

4. (a) The random variables X and Y have a joint probability mass function given by $P(X = x, Y = y) = \frac{x^2 + y^2}{32}$ for $x = 0, 1, 2, 3$ and $y = 0, 1$. Find the marginal probability mass function of X and Y. Also find $P(X \le 2, Y = 1)$.

(b) An urn always contains two balls of colours red and blue. At each stage a ball is randomly chosen and then replaced by a new ball, which with probability 0.8 in case of same colour replacement and with probability 0.2 in case of opposite colour replacement. If initially both the balls are red, find the probability that the 3rd ball selected is red.

6 + 6 = 12

5.(a) The joint distribution of X and Y is given by

$$f(x,y) = \begin{cases} e^{-(x+y)}, & x \ge 0, y \ge 0 \\ 0, & otherwise \end{cases}$$

Find the marginal density function of X and Y. Also calculate $P(X + Y \le 4)$.

(b) (i) A communication source can generate one of three possible messages 1, 2 and 3. Assume that the generation can be described by the Markov Chain with following transition probability matrix:

Current message	Next message		
	1	2	3
1	0.5	0.3	0.2
2	0.4	0.2	0.4
3	0.3	0.3	0.4

and the initial state probability distribution is $\pi^{(0)} = (0.3, 0.3, 0.4)$. Find the state probability distribution of third communication.

(ii) State Chapman-Kolmogorov equation.

(6) + (4 +2) = 12

Group – D

6.(a) Find the mean, variance and the co-efficient of skewness of the continuous distribution with probability density function given by,

$$f(x) = \begin{cases} 1 - |1 - x|, & 0 < x < 2 \\ 0, & elsewhere \end{cases}$$

(b) The following table provides data about the percentage of students who have free university meals and their CGPA scores. Calculate the Spearman's Rank Correlation between the two and interpret the result.

State University	Pune	Chennai	Delhi	Kanpur	Goa	Indore	Guwahati
% of students having free meals	14.4	7.2	27.5	33.8	38.0	15.9	4.9
% of students scoring above 8.5 CGPA	54	64	44	32	37	68	62

(2+ 2 + 3) + 5 = 12

7. (a) If X be a Normal (m, σ) variate, then prove that, $\mu_{2r+2} = \sigma^2 \mu_{2r} + \sigma^3 \dfrac{d\mu_{2r}}{d\sigma}$.

Hence, find the co-efficient of kurtosis of this distribution.

(b) Find the moment generating function of the exponential distribution. Use the function to find the mean and variance.

6 + 6 = 12

Group – E

8. (a) The life of an electronic device is normally distributed with mean 4 and variance 6. Ten devices are drawn at random in different ways. Find the probability that sample variance lies between 1.995 and 10.146. Also find the mean of the sampling distribution of sample variance.

[Given, $\chi^2_{0.950;9} = 3.325$; $\chi^2_{0.05;9} = 16.91$].

(b) (i) If T_1 and T_2 be statistic with expectation $E(T_1) = 2\theta_1 + 3\theta_2$ and $E(T_2) = \theta_1 + \theta_2$, find unbiased estimators of the parameters θ_1 and θ_2.

(ii) If the sample observations are 2, 4, 6, 8, 10 from an infinite population with variance σ^2, determine an unbiased estimate of σ^2.

(6 + (3+3)) = 12