DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Solution of the recurrence relation $a_n = 2a_{n-1}$ with $a_0 = 1$ is $S_n =$
(a) 2^n (b) 2^{n-1}
(c) 2^{n+11} (d) 2.

(ii) The number of bit strings of length 8 that begin with 1011 are
(a) 8 (b) 16
(c) 32 (d) 64.

(iii) The chromatic number of the cycle having seven vertices is
(a) 2 (b) 3
(c) 4 (d) 1.

(iv) The chromatic polynomial of a tree having six vertices is
(a) $x(x-1)^2$ (b) $x(x-1)^3$
(c) $x^6 - 1$ (d) $x(x-1)^5$.

(v) Which of the following graphs is non-planar?
(a) K_4 (b) K_3
(c) K_6 (d) C_6.

(vi) If a planar graph determines 10 regions, then the number of vertices of its dual is
(a) 8 (b) 9
(c) 10 (d) 11.

(vii) If a graph has three vertices and no edges, then its chromatic number is
(a) 2 (b) 3
(c) 0 (d) 1.

(viii) $\sim(\sim(\sim p)) \equiv$
(a) p (b) $\sim\sim p$
(c) Tautology (d) $\sim p$.

(ix) Converse of $p \to \sim q$ is
(a) $p \to q$ (b) $\sim q \to \sim p$
(c) $\sim q \to p$ (d) $\sim p \to q$.

(x) $\sim(\forall x \forall y P(x,y))$
(a) $\exists x \forall y \sim P(x,y)$ (b) $\exists x \exists y P(x,y)$
(c) $\sim(\exists x \forall y P(x,y))$ (d) $\exists x \exists y \sim P(x,y)$.

Group – B

2. (a) Using the truth table show that $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$.

(b) If p : Today is Friday.
q : It is raining.
r : It is hot.
Write the statement against the following symbol
(i) $\sim q \to (r \wedge p)$
(ii) $(p \vee q) \leftrightarrow r$
(iii) $(p \wedge \sim q) \to \sim r$.

6 + (2 + 2 + 2) = 12

3. (a) Find whether the conclusion C follows from the premises H_1, H_2, H_3 in the following case, using the truth table:
$H_1: p \vee q, H_2 : p \to r, H_3 : q \to r, C : r$.

(b) Show that $\{(p \wedge \sim q) \to r\} \to \{p \to (q \vee r)\}$ is a tautology.

6 + 6 = 12

Group – C

4. (a) A man buys 3 cows, 2 pigs and 4 hens from another person who has 7 cows, 6 pigs and 8 hens. How many choices does the man have?

(b) Let $U = \{1, 2, 3, \ldots\ldots 1000\}$. Then find $n(S)$ where, $S =$ set of such integers of U which are not divisible by 3, 5 or 7.

4 + 8 = 12

5. (a) Using characteristic root method find the solution of the recurrence relation $t_n + 9t_{n-2} = 6t_{n-1}$ subject to the initial condition $t_0 = 1$ and $t_1 = 6$.

(b) Using generating function solve the recurrence relation, $a_n - 7a_{n-1} + 10a_{n-2} = 2 \forall n > 1$ and $a_0 = 3$, $a_1 = 3$.

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Group – D

6. (a) Find the chromatic polynomial of K_6, the complete graph having 6 vertices. Show your calculations and justify your answer.

   (b) Let G be a regular graph, the degree of each of its vertices being 4. Determine the number of vertices of G if G determines 10 regions.

6 + 6 = 12

7. (a) Prove the following result:
   If a graph G has at least one edge then the sum of the coefficients in its chromatic polynomial is 0.

   (b) Let C_n denote the graph that is a cycle having n vertices. Prove that the chromatic number of C_n is
   (i) 2 if n is even.
   (ii) 3 if n is odd.

6 + 6 = 12

Group – E

8. (a) State Kuratowski's theorem. Use it to prove that K_6 is non-planar and K_4 is planar.

   (b) Prove Euler's Formula: A connected planar graph G with n vertices and e edges determines $f = e - n + 2$ regions.

6 + 6 = 12

9. (a) Prove that the chromatic polynomial of a having n vertices is $x(x-1)^{n-1}$.

   (b) Prove that the chromatic number K_n, the complete graph having n vertices is n.

6 + 6 = 12