MATHEMATICS-I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The degree and order of the differential equation $(\frac{d^2y}{dx^2}+2)^{3/2}=x\frac{dy}{dx}$ are
(a) degree= 3/2, order = 2 (b) degree = 2, order = 3
(c) degree = 3, order = 2 (d) degree = 2, order = 1.

(ii) If one of the eigenvalues of a matrix A is zero, then A is
(a) non-singular (b) singular
(c) orthogonal (d) identity.

(iii) The sequence $\{(-1)^n\}$ is
(a) convergent (b) monotonic increasing
(c) monotonic decreasing (d) oscillatory.

(iv) If A is a square matrix , then $A-A^t$ is
(a) symmetric matrix (b) skew-symmetric matrix
(c) identity matrix (d) rectangular matrix.

(v) The value of b for which $\vec{A}=\left(bx^2y+yz\right)\hat{i}+\left(xy^2-xz^2\right)\hat{j}+\left(2xyz-2x^2y^2\right)\hat{k}$ is solenoidal is
(a) -2 (b) 2 (c) 4 (d) -4.

(vi) The function $f(x,y)=\frac{1}{(\sqrt{x}+\sqrt{y}+\sqrt{z})}$ is a homogeneous function of degree
(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) 1.

(vii) If $D\equiv\frac{d}{dx}$, then $\frac{1}{D-a}X=$
(a) $\int Xe^{-ax}dx$ (b) $e^{-ax}\int Xe^{ax}dx$
(c) $e^{ax}\int Xe^{-ax}dx$ (d) $e^{-ax}\int Xe^{-ax}dx$.

B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/1ST SEM/MATH 1101/2019

(viii) If $AA^t=I$, then
(a) $\det A=0$ (b) $\det A=2$
(c) $\det A=\pm 1$ (d) $\det A=-2$.

(ix) If C is the circle $x^2+y^2=1$, then $\int_C(xdx+ydy)$ is
(a) 1 (b) 0 (c) 2 (d) 3.

(x) The series $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+\cdots\infty$ is
(a) convergent (b) divergent
(c) absolutely convergent (d) oscillatory.

Group – B

2. (a) Find the rank of the given matrix for different values of a:

$$\begin{bmatrix} a & -1 & 1 \\ -1 & a & -1 \\ -1 & -1 & a \\ 1 & 1 & 1 \end{bmatrix}$$

(b) If λ is a non zero eigen value of a non-singular matrix A, then prove that $\frac{1}{\lambda}$ is an eigen value of A^{-1}.

6 + 6 = 12

3. (a) If A is a real square matrix and $(I-A)(I+A)^{-1}$ is orthogonal, prove that A is skew-symmetric. I being the identity matrix of the same order as that of A.

(b) Find the eigen values and corresponding eigen vectors of the matrix

$$A=\begin{pmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{pmatrix}.$$

5 + 7 = 12

Group – C

4. (a) Test the convergence of the series: $\sum_{n=0}^{\infty}\frac{2^2.4^2.6^2...(2n)^2}{3.4.5....(2n+1)(2n+2)}x^{2n+2}$.

(b) Prove that if $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ and $r=|\vec{r}|$ then $curl\frac{\vec{r}}{r}=\vec{0}$.

6 + 6 = 12

5. (a) Test the convergence of the series $\sum \frac{4.7.10......(3n+1)}{1.2.3......n} x^n$.

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$.

6 + 6 = 12

Group – D

6. (a) Solve the following differential equation by D-operator method:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 4x.$$

(b) Solve the following differential equation:

$$(x^2+y^2+x)dx + xydy = 0.$$

6 + 6 = 12

7. (a) Solve: $p = \tan\left(x - \frac{p}{1+p^2}\right)$,where, $p = \frac{dy}{dx}$.

(b) Solve the following differential equation by the method of variation of parameters:

$$\frac{d^2y}{dx^2} + a^2y = \sec ax.$$

6 + 6 = 12

Group – E

8. (a) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$, prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$.

(b) Evaluate $\oint_C (x^2+xy)dx - (x^2-y)dy$ taken in the clockwise direction along the closed curve C formed by $y=x^2$ and $y=x$.

6 + 6 = 12

9. (a) Evaluate: $\int_0^a \int_0^x \int_0^y x^3y^2z\, dx\, dy\, dz$.

(b) Apply Green's theorem to evaluate $\int_C\{(2x^2-y^2)dx + (x^2+y^2)dy\}$ where, C is the boundary of the area enclosed by the x- axis and the upper-half of the circle $x^2 + y^2 = a^2$.

6 + 6 = 12