9. (a) The probability density function of a random variable X is $f(x) = k(x-1)(2-x)$ for $1 \leq x \leq 2$. Determine
   (i) The value of k;
   (ii) $P(\frac{5}{4} \leq X \leq \frac{3}{2})$.

   (b) A and B play a game in which their chances of winning are in the ratio $3:2$. Find A's chances of winning at least three games out of the five games played.

(3 + 3) + 6 = 12

NUMERICAL AND STATISTICAL METHODS (MATH 2002)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. The probability of getting at least one head is
   (a) $\frac{1}{32}$ (b) $\frac{13}{32}$ (c) $\frac{16}{32}$ (d) $\frac{31}{32}$

   (ii) What is the probability that a leap year selected at random, will contain 53 Saturdays?
   (a) $\frac{1}{7}$ (b) $\frac{3}{7}$ (c) 1 (d) $\frac{2}{7}$

   (iii) A continuous random variable X has a probability density function $f(x) = e^{-x}, 0 < x < \infty$. Then $P(X > 1)$ is
   (a) 0.368 (b) 0.5 (c) 0.632 (d) 1.0

   (iv) Let X be a random variable. Then which of the combinations of $E(X)$ and $E(X^2)$ respectively is not possible for the random variable X ?
   (a) 0 and 1 (b) 2 and 3 (c) $\frac{1}{2}$ and $\frac{1}{3}$ (d) 2 and 5.

   (v) If $u = 2x + 5$ and $v = 3y - 5$, and the correlation coefficient between x and y is 0.75, then the correlation coefficient between u and v is
   (a) 0.86 (b) 0.75 (c) –0.75 (d) 1.

   (vi) One of the roots of $x^3 - 17x + 5 = 0$ lies between
   (a) 1 and 2 (b) 0 and 1 (c) –1 and 0 (d) 6 and 7.

   (vii) $\Delta^2(e^x)$ (taking $h = 1$) is equal to
   (a) $(e-1)e^x$ (b) $(e-1)^2 e^{2x}$ (c) $(e-1)^2 e^x$ (d) e^x

(viii) In the trapezoidal rule for finding $\int_a^b f(x)dx$, $f(x)$ is approximated by
(a) a line segment (b) a parabola
(c) a circular sector (d) a part of an ellipse.

(ix) Let $f(x)$ be an interpolation polynomial. If $f(0) = 12$, $f(3) = 6$ and $f(4) = 8$, then $f(x)$ is
(a) $x^2 - 3x + 12$ (b) $x^2 - 5x$
(c) $x^3 - x^2 - 5x$ (d) $x^2 - 5x + 12$.

(x) In the LU-factorization method, a matrix A is factorized as $A = LU$ where, U is
(a) an upper triangular matrix (b) a lower triangular matrix
(c) the identity matrix (d) a diagonal matrix.

Group – B

2. (a) Find the smallest positive root of the equation $3x^3 - 9x^2 + 8 = 0$ correct to four places of decimal, using Newton-Raphson method.

(b) Find the root of the equation $x \tan x = 1.28$ that lies in the interval $(0, 1)$, correct to two places of decimal, using bisection method.

6 + 6 = 12

3. (a) Solve the following system of linear equations by Gauss elimination method.

$$x - 2y + 9z = 8,$$
$$3x + y - z = 3,$$
$$2x - 8y + z = -5.$$

(b) Using Gauss-Seidel method find the solution of the following system of linear equations correct to two places of decimal.

$$3x + y + 5z = 13,$$
$$5x - 2y + z = 4,$$
$$x + 6y - 2z = -1.$$

6 + 6 = 12

Group – C

4. (a) Using Lagrange's interpolation formula, find the polynomial of degree ≤ 3 passing through the points $(-1, 1)$, $(0,1)$, $(1, 1)$ and $(2, -3)$.

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using the trapezoidal rule. Hence compute an approximate value of π.

6 + 6 = 12

5. (a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition $y = 1$ at $x = 0$, find y for $x = 0.1$ by Euler's method, correct to four places of decimal, taking step length, $h = 0.02$.

(b) Find $y(1.1)$ using Runge-Kutta method of fourth order, given that $\frac{dy}{dx} = y^2 + xy$, $y(1) = 1$.

6 + 6 = 12

Group – D

6. (a) There are two identical urns containing respectively 4 white, 3 red balls and 3 white, 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball drawn is white, what is the probability that it is from the first urn ?

(b) A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is (i) an ace (ii) a heart (iii) neither a spade nor a ten.

6 + (2 + 2 + 2) = 12

7. (a) Two urns contain respectively 5 white, 7 black balls and 4 white, 2 black balls. One of the urns is selected by the toss of a fair coin and then 2 balls are drawn without replacement from the selected urn. If both balls are white, what is the probability that the first urn was selected.

(b) If A and B are independent events, show that the following pairs are independent.
(i) A and B^C
(ii) A^C and B
(iii) A^C and B^C

6 + (2 + 2 + 2) = 12

Group – E

8. (a) Calculate the median and mode of the following frequency distribution:

Marks:	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69
Frequency:	8	11	15	18	16	7

(b) From the following data, obtain the line of regression of X on Y.

X:	91	97	108	121	67	73
Y:	71	75	69	97	70	61.

6 + 6 = 12