(ii) Prove that the reciprocal lattice vector $\vec{G} = h\,\vec{b_1} + k\,\vec{b_2} + l\,\vec{b_3}$ is perpendicular to the (hkl) plane of a crystal lattice, where $\vec{b_1}$, $\vec{b_2}$ and $\vec{b_3}$ are axis vectors of the reciprocal lattice. Prove that the distance between two adjacent parallel planes of the lattice is $2\pi/|\vec{G}|$.

(b) What are the two major types of point defects in crystalline materials?

(c) Show that the reciprocal lattice for a simple cubic structure is another simple cubic.

(d) Explain Ewald construction with diagram.

[2 + (2 + 2)] + 1 + 2 + 3 = 12

Group – E

8. (a) Sketch approximately the phonon dispersion curves for a CsCl crystal. Figure out the value of ω in two branches at $k = 0$ and at the boundary of the first Brillouin zone.

(b) What is meant by the first Brillouin zone? Explain why the phonon dispersion curves are only drawn for the first Brillouin zone.

(c) The group velocity in a certain linear monatomic chain at small k is 1.08×10^4 m/s. If the mass of each atom is 6.81×10^{-26} kg and the atomic separation at equilibrium is 0.485 nm, find (i) the effective spring constant and (ii) the maximum normal mode angular frequency.

(2 + 4) + (2 + 1) + (1 + 2) = 12

9. (a) The energy-wave vector dispersion relation for a one dimensional crystal of lattice constant 'a' is given by $E(\kappa) = E_0 + 2\alpha\kappa^2 - 3\beta\kappa^4$, where E_0, α, β are positive constants. Find the expression for the velocity of the electron as a function of κ. For what value of κ the velocity is the maximum?

(b) The relation between the Bloch wave vector K and free particle wave vector k for typical Kronig-Penny model is given by

$\cos Ka = \cos ka + \frac{\Omega}{k}\sin ka$.

Explain the existence of band structure in one dimension in view of this relation.

(c) The so called E-k relation in a band structure is given by $E = a + bk^2$. Find the group velocity of a carrier. Determine its effective mass.

(d) State Bloch theorem for an electron in a periodic lattice in one dimension.

(2 + 2) + 3 + (2 + 2) + 1 = 12

PHYSICS - II
(PHYS 2111)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The eigen value of the energy of a particle in a cubical box of dimension 'a' is $\left[11\left(\frac{h^2}{8ma^2}\right)\right]$. Possible quantum numbers of the state are

(a) (3, 1, 1) (b) (3, 0, 1) (c) (2, 2, 2) (d) (1, 1, 1)

(ii) Which of the following functions are Eigen functions of the operator $\frac{d^2}{dx^2}$?

(a) $\psi = c\log x$ (b) $\psi = cx^2$ (c) $\psi = \frac{c}{x}$ (d) $\psi = ce^{-mx}$

[where c and m are arbitrary constants]

(iii) The pair of observables that cannot be measured simultaneously is

(a) $\hat{p}_x, \hat{p}_y$ (b) $\hat{x}, \hat{p}_y$ (c) $\hat{y}, \hat{p}_y$ (d) $\hat{x}, \hat{y}$

(iv) Which one of the following equations are true for a perfect conductor?

(a) $\frac{\partial \vec{j}}{\partial x} = \frac{ne^2}{m}\vec{E}$ (b) $\frac{\partial \vec{j}}{\partial t} = \frac{ne^2}{m}\vec{E}$

(c) $\frac{\partial \vec{j}}{\partial t} = \frac{ne^2}{m}\vec{B}$ (d) $\frac{\partial \vec{B}}{\partial t} = \frac{ne^2}{m}\vec{j}$

(v) Number of lattice points in a bcc iron unit cell is

(a) 1 (b) 2 (c) 3 (d) 4.

(vi) Dimension of an axis of a reciprocal lattice vector is

(a) [L] (b) $[L]^{-1}$ (c) dimensionless (d) $[L]^{-2}$

(vii) The group velocity of an electron from a dispersion relation inside a solid material is given by

(a) $v_g = \frac{1}{h}\frac{dE}{dk}$ (b) $v_g = \frac{2\pi}{h}\frac{dE}{dk}$

(c) $v_g = h\frac{dE}{dk}$ (d) $v_g = \frac{h}{2\pi}\frac{dE}{dk}$

(viii) Potential energy of a magnetic dipole in a magnetic field is given by

(a) $u = -\vec{\mu}_m^0.\vec{B}$ (b) $u = |\vec{\mu}_m^0 \times \vec{B}|$

(c) $u = \vec{\mu}_m^0.\vec{B}$ (d) None of the above.

(ix) The energy of an elastic mode of angular frequency ω is

(a) (n+ ½) ℏω (b) (n+ ½) hω (c) nhω (d) nℏω.

(x) The span of first Brillouin zone of a crystal vibration with monatomic lattice having lattice constant 'a' is

(a) $-\pi \le k \le \pi$ (b) $-\frac{\pi}{a} \le k \le \frac{\pi}{a}$

(c) $-\frac{\pi}{2} \le k \le \frac{\pi}{2}$ (d) $-\frac{\pi}{2a} \le k \le \frac{\pi}{2a}$

Group – B

2. (a) What are the basic postulates of quantum mechanics?

(b) Obtain the expression for the eigen function of the momentum operator $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ corresponding to an eigen value p_x.

(c) Show that for a particle in a rigid box (spanning from $x = 0$ to $x = a$) the eigen function is given by $\psi(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$. Also find the energy eigen values.

(d) Compute the lowest energy of an electron confined within a cubical box of size of 10^{-14} m. $(h = 6.62\times10^{-34}\,Js,\ m = 9.1\times10^{-31}\,kg)$

2 + 3 + (3 + 2) + 2 = 12

3. (a) A particle exists inside a one dimensional potential box of length L. Calculate the probability of finding the particle in a region between $\frac{L}{4}$ and $\frac{3L}{4}$ when the particle is in the lowest energy state.

(b) Using the principle of operator correspondence write down the operator $\hat{L}_x, \hat{L}_y, \hat{L}_z$ (components of angular momentum) in terms of position and momentum operators. Then show that $[\hat{L}_x, \hat{L}_y] = i\hbar\hat{L}_z$.

(c) Consider $\varphi = c_1\varphi_1 + c_2\varphi_2$, where φ_1 and φ_2 are orthonormal energy eigenstates of a system corresponding to energy E_1 and E_2 at $t = 0$. If φ is normalized and $c_1 = \frac{1}{\sqrt{3}}$ then what is the value of c_2? Find the expectation value of E^2.

3 + (3 + 3) + (1 + 2) = 12

Group – C

4. (a) State Weiss hypothesis for ferromagnetic materials.

(b) Using Weiss hypothesis show that the susceptibility of a ferromagnetic substance in its paramagnetic phase is inversely proportional to its temperature.

(c) Applying Hund's rule and Pauli's exclusion principle show that in the ground state the Carbon atom does not have permanent magnetic dipole moment.

(d) Draw the B-H curves for soft magnet, hard magnet and paramagnet.

2 + 4 + 3 + (1 + 1 +1) = 12

5. (a) What is the critical magnetic field for a superconductor? How does it vary with temperature? Show the variation graphically.

(b) Cadmium (Cd) gets transition to its superconducting state at 0.52 K. If its critical magnetic field at 0 K is 0.0028 T, calculate its critical magnetic field at -271°C.

(c) The ends of a normal metal wire are subject to a constant potential difference.

(i) Write down the fundamental equation of motion of the free electrons inside the wire.

(ii) Solve the equation to find the mean velocity of an electron as a function of time.

(iii) Show that in the steady state the current density is proportional to the electronic density *n* and to the electric field $\vec{E}$.

(2 + 1 + 1) + 2 + (2 + 2 + 2) = 12

Group – D

6. (a) Show (120) and (11$\bar{1}$) planes in Cartesian system by drawing a unit cell in each case.

(b) What is primitive unit cell? Write down the primitive basis vectors of bcc lattice. Find out the angle between two primitive basis vectors of bcc lattice.

(c) Find out *h*, *k*, *l* values for (hkl) planes with an interplanar spacing of 1.811 Å in cubic copper, having *a* = 2.561 Å.

(2 + 2) + (2 + 2 + 2) + 2 = 12

7. (a) (i) Obtain Bragg condition from $2\,\vec{k}.\vec{G} = G^2$, the symbols have their usual meanings.