MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The only function amongst the following that is analytic on the entire complex plane is

(a) $\frac{1}{z-1}$ (b) cosec z (c) sec z (d) e^z.

(ii) Equation $p\tan y + q\tan x = sec^2 z$ (where, $= \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$) is of order

(a) 1 (b) 2 (c) 0 (d) 3.

(iii) The Fourier transform of $f(\mathrm{t}) = 5e^{-3|t-1|}, t \in \mathbb{R}$ is

(a) $\frac{30}{9+s^2}e^{-is}$ (b) $\frac{30}{9+s^2}$ (c) $\frac{6}{9+s^2}$ (d) $\frac{6}{9+s^2}e^{-is}$

(iv) If $f(x) = \pi - x, 0 < x < \pi$ be represented in Fourier series as $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then the value of a_0 is

(a) 2 (b) 0 (c) 4 (d) 1.

(v) The solution of $\sqrt{p} + \sqrt{q} = 1$ is (where, $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$)

(a) $z = ax + \left(1-\sqrt{a}\right)^2 y + c$ (b) $z = ax + \left(1-\sqrt{a}\right)^2 y$

(c) $z = ax^2 + \left(1-\sqrt{a}\right)^2 y$ (d) $z = ax^2 - \left(1-\sqrt{a}\right)^2 y + c$

(vi) The complementary function of $4r - 4s + t = 16\log(x+2y)$ is (where, $r = \frac{\partial^2 z}{\partial x^2}, \mathrm{s} = \frac{\partial^2 z}{\partial x \partial y}$ and $t = \frac{\partial^2 z}{\partial y^2}$)

(a) $f_1\left(y + \frac{1}{2}x\right) + f_2\left(y - \frac{1}{2}x\right)$ (b) $f_1\left(y + \frac{1}{2}x\right) + x\, f_2\left(y + \frac{1}{2}x\right)$

(c) $f_1\left(y + \frac{1}{2}x\right)$ (d) $x\, f_1\left(y + \frac{1}{2}x\right) + x^2 f_2\left(y + \frac{1}{2}x\right)$

(vii) The singular points of the equation $x(x-1)y'' + x^2y' + y = 0$ are

(a) 1,0 (b) 2,3

(c) 3,4 (d) −2, −3.

(viii) The Fourier transform of $f(x) = \begin{cases} 1, -1 < x < 1 \\ 0, \, otherwise \end{cases}$ is

(a) $\sin s$ (b) $\frac{\sin s}{s}$ (c) $\frac{\cos s}{s}$ (d) $\frac{2\sin s}{s}$

(ix) The value of $\int_C \frac{dz}{(z-50)^{50}}$ where $z = 50$ is an interior point of C is

(a) 50 (b) 50 πi (c) 0 (d) 2 πi.

(x) $P_n(-x) =$

(a) $P_n(x)$ (b) $(-1)^n \ P_n(x)$ (c) $-P_n(x)$ (d) $P_{-n}(x)$.

B.TECH/AEIE/CE/ECE/EE/3RD SEM/MATH 2001 (BACKLOG)/2019

Group – B

2. (a) Prove that the function defined by $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at the origin.

(b) Evaluate the integral $\int_C \frac{z}{z^2-25} dz$ on the circle $C: |z-1| = 2$. (1,0) is the centre and 2 is radius of the circle C.

6 + 6 = 12

3. (a) Prove that $u(x, y) = 2x - 2xy$ is a harmonic function. Determine its harmonic conjugate.

(b) Evaluate $\oint_C \frac{3z^2 + z - 1}{(z^2-1)(z-3)} dz$ around the circle $C: |Z| = 2$, using Cauchy's residue theorem.

6 + 6 = 12

Group – C

4. (a) Find the half-range Fourier cosine series of the function $f(x) = x, 0 \le x \le 2$. Hence, using Parseval's identity, prove that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + ... = \frac{\pi^4}{96}$.

(b) Find the Fourier Transform of the function $f(x) = \begin{cases} 1, |x| < 1 \\ 0, |x| > 1 \end{cases}$.

Represent $f(x)$ as a Fourier Integral and hence find the value of $\int_0^\infty \frac{\sin\alpha\cos\alpha}{\alpha}\,d\alpha$.

5 + 7 = 12

5. (a) Find the half range cosine series of the function $f(x) = x(\pi - x)$ in $0 < x < \pi$ and using Parseval's identity also prove that $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$

   (b) Find the Fourier transform of $f(x) = e^{-a|s|}$.

7 + 5 = 12

Group – D

6. (a) Solve in series the equation $(1 + x^2)y'' + xy' - y = 0$ about the point $x = 0$.

   (b) Show that $P_n(1) = 1$.

8 + 4 = 12

7. (a) Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

   (b) Show that $J_{-n}(x) = (-1)^n J_n(x)$, where, n is a positive integer.

6 + 6 = 12

Group – E

8. (a) Form the partial differential equation by eliminating the arbitrary function f from $f\left(xy + z^2,\ x + y + z\right) = 0$.

   (b) Solve by the method of separation of variables: $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, where, $u(0, y) = 8e^{-3y}$.

6 + 6 = 12

9. (a) Find the general solution of the following differential equation $(D^2 - 7DD' + 12D'^2)z = e^{x-y}$, where, $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$.

   (b) Solve the given linear partial differential equation by Lagrange's method: $(mz - ny)p + (nx - lz)q = ly - mx$, where, $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

6 + 6 = 12