9. (a) For two variables x and y, the equations of two regression lines are $4x + y = 56$ and $x + y = 32$. Identify which one is the regression line of y on x. Find the correlation coefficient between x and y. Find the mean of x and the mean of y. Estimate σ_x given that $\sigma_y = 2$.

(b) The expenditure of 1000 families is given below:

Expenditure (Rs.)	40 – 59	60 – 79	80 – 99	100 – 119	120 – 139
Frequency	50	f_2	500	f_4	50

The median and mean for the distribution are both Rs. 87.50. Calculate the missing frequencies.

6 + 6 = 12

B.TECH/BT/3RD SEM/MATH 2101/2019

MATHEMATICAL & STATISTICAL METHODS
(MATH 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In Trapezoidal rule to evaluate $\int_a^b f(x)dx$, the curve $y = f(x)$ is approximated by
(a) straight line (b) parabola
(c) exponential function (d) hyperbola.

(ii) Particular integral of the PDE $(D^2 + 2DD' + D'^2)z = (2x + 3y)$ is
(a) $\frac{1}{150}(2x+3y)^3$ (b) $\frac{1}{75}(2x+3y)^3$
(c) $\frac{1}{150}(2x+3y)^2$ (d) $\frac{1}{75}(2x+3y)^2$.

(iii) A random variable X follows Gamma distribution with parameter α and β then $E(X)$ is
(a) $\alpha\beta$ (b) α (c) β (d) $\frac{\alpha}{\beta}$.

(iv) The value of a_0 in the Fourier series of $f(x) = x^3$ in $(-3, 3)$ is:
(a) $\frac{81}{2}$ (b) $\frac{1}{3}$ (c) 162 (d) 0.

(v) $(\Delta - \nabla)x^2$ is equal to
(a) $2h^2$ (b) $3h^2$ (c) $4h^2$ (d) $\frac{h^2}{2}$.

(vi) If $u = 2x + 5$ and $v = -3y + 1$ and regression coefficient of y on x is -1.2, then the regression coefficient of v on u is
(a) 1 (b) −1.2 (c) 1.2 (d) 1.8.

(vii) Equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$ is of order
(a) 1 (b) 2 (c) 3 (d) 4.

(viii) Quartile deviation of $24, 7, 11, 9, 17, 3, 20, 14, 14, 4, 22, 27$ is
(a) 7 (b)7 (c) 22 (d) 9.

(ix) The (n +1)th order forward difference of a polynomial of degree n is
(a) *n!* (b) (n +1)! (c) 0 (d) (n -1)!.

(x) The equation $Pp + Qq = R$ is known as
(a) Charpit's equation (b) Lagrange's equation
(c) Bernoulli's equation (d) Clairaut's equation

Group – B

2. (a) Form a partial differential equation from the relation $f(x^2 + y^2, z - xy) = 0$ by eliminating the arbitrary function *f*.

(b) Solve the linear partial differential equation $(y+z)p+(z+x)q=(x+y)$.

6 + 6 = 12

3. (a) Solve the following partial differential equation $\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 12\frac{\partial^2 z}{\partial y^2} = e^{x-y}$ by D-operator method.

(b) Solve by the method of separation of variables: $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where, $u(x,0) = 6e^{-3x}$.

6 + 6 = 12

Group – C

4. (a) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$ taking $h = 1$.

(b) Find the missing term in the following table:-

x	2	4	6
y	5.6	–	13.9

(c) Given the function $y = \frac{1}{x}$, show that the divided difference of n^{th} order is $y[x_0, x_1, x_2, \dots, x_n] = \frac{(-1)^n}{x_0 x_1 x_2 \dots x_n}$.

3 + 3 + 6 = 12

5. (a) Use Trapezoidal rule to estimate the integral $\int_0^2 e^{x^2} dx$ taking 10 intervals.

(b) Find the volume of the tapered wood whose radii of cross sections from one end are given below:

x	0	5	10	15	20	25
Y	4	6	7	10	7	2

6 + 6 = 12

Group – D

6. (a) Find the Fourier series of the function $f(x) = x^2 - 2$ in the interval $(-2, 2)$.

(b) Find the half-range cosine series for $f(x) = (x-1)^2, 0 < x < 1$. Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$.

6 + 6 = 12

7. (a) Using Parseval's Identity for half-range sine series of the function $f(x) = x(\pi - x)$ in $0 < x < \pi$, prove that $\frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \dots = \frac{\pi^6}{960}$.

(b) From the Fourier series expansion of $f(x) = x^2$ in $-\pi < x < \pi$, prove that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$.

6 + 6 = 12

Group – E

8. (a) A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is Poisson distributed with the average number of demand per day being 1.5. Calculate the proportion of days on which neither of the car is used and the proportion of days on which some demand is refused.

(b) The length of shower in a tropical island during rainy season follows an exponential distribution with parameter $\lambda(>0)$, where, time is measured in minutes. Find the probability that it will last for at least one more minute.
100 unbiased coins are tossed. Using normal approximation to binomial distribution calculate the probability of getting
(i) exactly 40heads.
(ii) 55 heads or more.

4 + 3 + 5 = 12