OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES (MATH 4181)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The set S given by $S = \{(x, y): x^2 + y^2 \leq 25,\ x, y \in \mathbb{R}\}$ is
(a) convex (b) open
(c) non-convex (d) unbounded.

(ii) Any solution to an L.P.P. (Linear Programming Problem) which satisfies the non-negativity restrictions of the L.P.P. is called its
(a) unbounded solution (b) optimal solution
(c) feasible solution (d) basic solution.

(iii) When total supply is equal to total demand in a transportation problem, the problem is said to be
(a) degenerate (b) unbalanced
(c) balanced (d) non-degenerate.

(iv) The feasible region of an L.P.P. is
(a) convex (b) non-convex
(c) open (d) unbounded.

(v) Assignment problem is solved by
(a) North West Corner Rule (b) Simplex Method
(c) Vogel's Approximation Method (d) Hungarian Method.

(vi) When the primal problem has a degenerate optimal solution, then the dual has
(a) degenerate solutions (b) infeasible solutions
(c) unbounded solutions (d) multiple optimal solutions.

(vii) In a fair game the value of the game is
(a) 1 (b) 0 (c) unbounded (d) 2.

(viii) The function $f(x,\ y) = x^2 - y^2$ has, at point $(0, 0)$
(a) global maximum (b) global minimum
(c) neither maximum nor minimum (d) local minimum.

(ix) The Hessian matrix of a function $f(x, y, z)$ is

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

If the function had a stationary point, this would be
(a) a global maximum point (b) a global minimum point
(c) a saddle point (d) a global maximum point.

(x) Which of the following elimination methods is most efficient to find the local optima of an unimodal function of one variable
(a) Golden section method (b) Fibonacci Method
(c) Interval Halving (d) Dichotomous Search.

Group – B

2. (a) Solve the following linear programming problem by graphical method:
Maximize $z = x + 2y$
Subject to

$$-x + 2y \le 8$$
$$x + 2y \le 12$$
$$x - y \le 3$$
$$x,\ y \ge 0$$

(b) Use Simplex method to solve the following linear programming problem:
Maximize $z = 60x_1 + 50x_2$
Subject to

$$x_1 + 2x_2 \le 40$$
$$3x_1 + 2x_2 \le 60$$
$$x_1, x_2 \ge 0$$

4 + 8 = 12

3. (a) Solve the following L.P.P. using Big-M method
Minimize $z = 2x_1 + 3x_2$
Subject to

$$3x_1 + 5x_2 \ge 30$$
$$5x_1 + 3x_2 \ge 60$$
$$x_1,\ x_2 \ge 0$$

(b) Write the dual of the following L.P.P.:
Maximize $z = x_1 + 5x_2$

Subject to

$$3x_1 + 4x_2 \leq 6$$
$$x_1 + 3x_2 \geq 2$$
$$x_1,\ x_2 \geq 0$$

9 + 3 = 12

Group – C

4. (a) Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method:

	W_1	W_2	W_3	W_4	**Capacity**
F_1	10	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

(b) A salesman has to visit five cities A, B, C, D and E. The distance (in hundred miles) between the five cities are as follows:

	A	B	C	D	E
A	-	7	6	8	4
B	7	-	8	5	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A, which route should he select so that the total distance travelled is minimum?

6 + 6 = 12

5. (a) Use dominance to reduce the following game problem to 2×2 game and hence find the optimal strategies and the value of the game

PLAYER **B**

PLAYER **A**	3	-2	4
	-1	4	2
	2	2	6

(b) Solve graphically the game whose payoff matrix is given by

PLAYER **B**

PLAYER **A**	0	-2
	7	-1
	-1	4
	-2	6
	5	-3

6 + 6 = 12

Group – D

6. (a) Find the stationary point(s) along with its nature for the function $f(x,y,z) = x^2 + 4y^2 + 4z^2 + 4xy + 4xz + 16yz$.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:
Minimize $f(x_1,x_2,x_3) = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$
Subject to the constraints

$$x_1 + x_2 + x_3 = 20$$
$$x_1, x_2, x_3 \geq 0$$

4 + 8 = 12

7. (a) Maximize $f(x_1,x_2) = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
Subject to the constraints

$$x_2 \leq 8$$
$$x_1 + x_2 \leq 10$$
$$x_1,\ x_2 \geq 0$$

by applying Kuhn-Tucker conditions.

(b) Show that the function $f(x_1,x_2) = x_1x_2 - x_1^2 - x_2^2$ is concave over $\mathbb{R}^2$.

10 + 2 = 12

Group – E

8. Write the Dichotomous Search Algorithm for unimodal functions of one variable and use the algorithm to find the minimum of $x(x - 1.5)$ in the interval $(0, 1)$ to within 10% of the exact value.

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9. Minimize the function

$$f(x) = 0.65 - [\tfrac{0.75}{1+x^2}] - 0.65\, x\, tan^{-1}\left(\tfrac{1}{x}\right)$$

using Golden Section method with $n = 6$.

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