SPECIAL SUPPLE B.TECH/CSE/AEIE/8TH SEM/ MATH 4282/2018

ADVANCED COMPUTATIONAL MATHEMATICS AND GRAPH THEORY (MATH 4282)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The smallest prime divisor of the number $2.3.5.7 + 1$ is
(a) 3 (b) 17
(c) 11 (d) none of the others.

(ii) The value of H_4, the fourth harmonic number, is
(a) 11/6 (b) 25/12
(c) 137/60 (d) none of the others.

(iii) The number of subsets of a set having n elements is
(a) $2^n - 1$ (b) 2^n
(c) $2n$ (d) $2^n - 2$.

(iv) The remainder in the division of $1! + 2! + 3! + 4! + \cdots + 20!$ by 4 is
(a) 2 (b) 3
(c) 1 (d) 0.

(v) The greatest common divisor of F_8andF_9 (i.e the 8th and 9th Fibonacci numbers) is
(a) 1 (b) 2
(c) 3 (d) 4.

(vi) Consider the following recurrence: $T_n = T_{n-1} + 4,\ T_0 = 3, n \geq 1.$ T_{100} is
(a) even (b) prime
(c) odd (d) negative.

(vii) Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. Then $\lfloor \sqrt[100]{20} \rfloor =$

(a) 2 (b) 3

(c) 4 (d) 1

(viii) The chromatic number of a bipartite graph is

(a) 3 (b) 2

(c) 1 (d) 0

(ix) The constant term of the chromatic polynomial of a graph having 6 vertices is

(a) 6 (b) 5

(c) -1 (d) 0.

(x) The number of perfect matchings in C_5, the cycle having five vertices, is

(a) 1 (b) 3

(c) 2 (d) 0.

Group - B

2. (a) Solve the following recurrence:

$$V_0 = 1\,;$$
$$V_n = 2V_{n-1}\,, \qquad for\ n > 0.$$

(b) Find V_2, V_3 and V_4. Show your work.

Consider the following recurrence.

$$K_0 = 1\,;$$
$$K_n = K_{n-1} + \ 2n\,, for\ n > 0.$$

Find an m such that (i) $K_m > 100$, (ii) $K_m > 1000$. Show your work.

6 + 6 = 12

3. (a) Calculate $\sum_{k=0}^{10}(2+3k)$ and $\sum_{k=0}^{100}(2+3k)$. Find an m such that $\sum_{k=0}^{m}(2+3k) > 300$. Show your work.

(b) Calculate $\sum_{k=0}^{100} 2.\,3^k$ and $\sum_{k=0}^{10} 2.\,(3^k + 3k)$. Show your work.

6 + 6 = 12

Group - C

4. (a) Let F_n denote the nth Fibonacci number.

The first few Fibonacci numbers are $F_0 = 0,\ F_1 = 1,\ F_2 = 1,\ F_3 = 2,\ F_4 = 3.$ Calculate $F_5, F_6, F_7, \ldots \ldots . . F_{20}$. Observe which ones are odd and which ones are even. Is F_{40} even or odd? Justify your answer and show your work.

(b) State the Binomial Theorem. Calculate $\binom{10}{0}+\binom{10}{1}+\binom{10}{2}+\binom{10}{3}+\ldots\ldots+\binom{10}{10}$ by using this theorem or otherwise. Show your work.

7 + 5 = 12

5. (a) The Stirling number of the second kind, $\left\{ n \atop k \right\}$, is the number of ways to partition a set of n things into k nonempty subsets. These numbers obey the recurrence relation

$$\left\{ n \atop k \right\} = k\left\{ n-1 \atop k \right\} + \left\{ n-1 \atop k-1 \right\}, integer\ n > 0.$$

Observe that $\left\{ n \atop 1 \right\} = 1, \left\{ n \atop n \right\} = 1$ for all $n > 0$. Calculate $\left\{ 3 \atop 2 \right\}, \left\{ 4 \atop 2 \right\}$ and $\left\{ 5 \atop 2 \right\}$ by using the recurrence relation or otherwise. Show your work.

(b) The Stirling number of the first kind, $\left[n \atop k \right]$ is the number of ways to arrange n objects into k cycles. By using this definition, Find the values of $\left[1 \atop 1 \right], \left[2 \atop 1 \right], \left[3 \atop 1 \right]$ and $\left[3 \atop 2 \right]$. Show your work.

6 + 6 = 12

Group - D

6. (a) State Fermat's Little Theorem. Show that
(i) $3^{16} \equiv 4(mod\ 7)$,
(ii) $3^{35} \equiv 5(mod\ 7)$ by using this theorem or otherwise.

(b) Find the greatest common divisor of 89 and 55 by using the Euclidean algorithm. Show your calculations in detail.

6 + 6 = 12

7. (a) By using Wilson's Theorem, or otherwise, find the remainder in the division of $18! + 7! + 6!$ by 19. Show your work.

(b) Let $\varphi(n)$ denote the Euler φ-function. Calculate (i) $\varphi(10)$ (ii) $\varphi(31)$ and (iii) $\varphi(105)$. Show your work.

5 + 7 = 12

Group - E

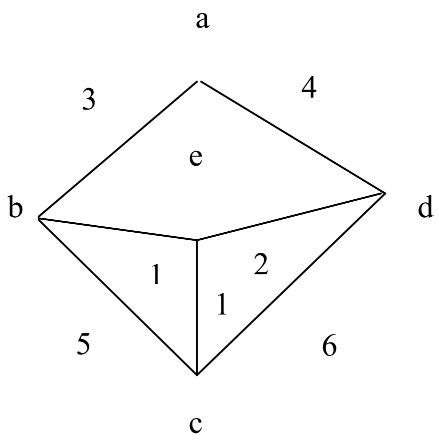
8. (a) State the definition of the chromatic polynomial of a graph. Find the chromatic polynomial of K_6, the complete graph having 6 vertices. Show your work.

(b) Find the chromatic number of (i) K_6, (ii) a tree and (iii) a graph having 4 vertices and no edges. Justify your answers.

5 + 7 = 12

9. (a) State the definition of a matching in a graph. Find
(i) all maximal matchings in K_4 , the complete graph having 4 vertices and
(ii) a perfect matching in C_6 , the cycle having 6 vertices.

(b) Find a spanning tree of the following graph by Kruskal's algorithm.



6 + 6 = 12