SPECIAL SUPPLE B.TECH/AEIE/CSE/7TH SEM/MATH 4182/2018

LINEAR ALGEBRA (MATH 4182)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The eigenvalues of the matrix $\begin{pmatrix} 3 & 17 & 26 \\ 0 & 15 & 39 \\ 0 & 0 & -2 \end{pmatrix}$ are

(a) 3,0,2 (b) 54,31,23 (c) 3,15,-2 (d) -6,9,13.

(ii) The singular values of $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ are

(a) 1,3 (b) 1,4 (c) 2,3 (d) 2,4.

(iii) Determine which of the following set of vectors is linearly dependent.
(a) {(1,-3,7), (2,0,-6), (3,-1,-1), (2,4,-5)} (b) {(1,0),(0,1)}
(c) {(1,2,-3), (1,-3,2), (2,-1,5)} (d) {sint, cost, t}.

(iv) If $u = (1, -2, k)$ is a linear combination of $(3,0,-2)$ and $(2,-1,-5)$ then the value of k is
(a) 2 (b) -5 (c) 1 (d) -8.

(v) Which of the following is a linear transformation?
(a) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by $T(x,y) = (x+1, 2y, x+y)$
(b) $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ defined by $T(x,y,z) = (|x|, 0)$
(c) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defined by $T(x,y) = (2x-y, x)$
(d) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}$ defined by $T(x,y) = |x+y|$

(vi) Let a, b and c be three vectors in $\mathbb{R}^n$. Which of the following relations is true?
(a) $\|a\| = \|b\|$ (b) $|\ a^T b\ | \leq \|a\|\|b\|$
(c) $\|a+c\| = \|b-c\|$ (d) $\|a-c\| \leq \|b-c\|$

(vii) If v = (1,2,-3) and w = (1,-4,3) in R^3. Then inner product $\langle v,w \rangle$ is

(a) 0 (b) 5 (c) -16 (d) 13.

(viii) The dimension of a vector space is

(a) the number of vectors in a basis
(b) the number of subspaces
(c) the dimension of its subspaces
(d) the number of vectors in a spanning subset.

(ix) Let the linear transformation $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be $T(x, y) = (x, x + y, y)$. Then dim Ker T is

(a) 0 (b) 1 (c) 2 (d) 3.

(x) A real quadratic form in three variables is $Q = x^2 + 2y^2 + 4z^2 + 2xy - 4yz - 2xz$. Then Q is

(a) positive semi-definite (b) indefinite
(c) negative definite (d) positive definite.

Group – B

2. (a) Prove: The characteristic equation of an orthogonal matrix P is a reciprocal equation.

(b) Find an orthogonal matrix P that diagonalizes the symmetric matrix

$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$

4 + 8 = 12

3. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & -4 \\ 0 & 5 & 4 \\ -4 & 4 & 3 \end{pmatrix}.$$

(b) Find the Singular Value Decomposition (SVD) of the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$.

6 + 6 = 12

Group – C

4. (a) Prove that the intersection of two subspaces of a vector space V over a field F is a subspace of V. Is the statement true for union of subspaces? Justify your answer.

(b) Determine the subspace of $\mathbb{R}^3$ spanned by the vectors $\alpha = (1,2,3)$, $\beta = (3,1,0)$. Examine if $\gamma = (2,1,3)$, $\delta = (-1,3,6)$ are in the subspace.

6 + 6 = 12

5. (a) Examine if the set S is a subspace of $\mathbb{R}^3$, where $S = \{(x,y,z) \in \mathbb{R}^3 : xy = z\}$.

(b) Prove that if the set $\{\alpha, \beta, \gamma\}$ is a basis of a real vector space V, then the set $\{\alpha + \beta + \gamma, \beta + \gamma, \gamma\}$ is also a basis of V.

6 + 6 = 12

Group – D

6. (a) Project the vector b = (3, 4, 4) onto the line through a = (2,2,1) and then onto the plane that also contains a*= (1,0,0).

(b) Find an orthogonal matrix P whose first row is (⅓, ⅔, ⅔).

6 + 6 = 12

7. (a) Apply Gram-Schmidt orthogonalization to the following set of vectors in $\mathbb{R}^3$: $\left\{\begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}8\\1\\-6\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$.

(b) If S is the subspace of $\mathbb{R}^3$ containing only the zero vector, what is $S^\perp$? If S is spanned by (1,1,1), what is $S^\perp$? If S is spanned by (2,0,0) and (0,0,3), what is a basis for $S^\perp$?

6 + 6 = 12

Group – E

8. (a) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by
$T(x,y,z) = (x + 2y - z, y + z, x + y - 2z)$.
Find a basis and the dimension of the image space of T.

(b) Prove that a linear transformation $T: V \longrightarrow W$ is injective iff null(T)={θ}

6 + 6 = 12

9. (a) Show that the following mapping is not linear: $T: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by $T(x,y,z) = (x + 1, y + 1, z + 1)$.

(b) Verify the rank-nullity theorem for $L: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$, where
$L\left(\begin{bmatrix}a\\b\\c\end{bmatrix}\right) = \begin{bmatrix}a - b + c\\-a + b - c\end{bmatrix}$.

5 + 7 = 12