**MCA/2ND SEM/MCAP 1204/2019**

**OPTIMIZATION TECHNIQUES**

**(MCAP 1204)**

**Time Allotted : 3 hrs Full Marks : 70**

***Figures out of the right margin indicate full marks.***

***Candidates are required to answer Group A and***

***any 5 (five) from Group B to E, taking at least one from each group.***

***Candidates are required to give answer in their own words as far as practicable.***

**Group – A**

**(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**

 (i) A feasible solution to an LP problem

(a) must satisfy all the problem’s constraints

(b) need not satisfy all the constraints ,only some of them

(c) must be a corner point of the feasible region

(d) must optimize the value of the objective function.

 (ii) If an optimal solution is degenerate, then

(a) there are alternative optimal solutions

(b) the solution is infeasible

(c) the solution is of no use to the decision-maker

(d) none of the above.

 (iii) To convert ≥ inequality constraints into equality constraints, we must

(a) add a surplus variable

(b) subtract an artificial variable

(c) subtract a surplus variable and add an artificial variable

(d) add a surplus variable and subtract an artificial variable.

 (iv) If an artificial variable is present in the “Basic Variable” column of the optimal simplex table, then the solution is

(a) infeasible (b) unbounded

(c) degenerate (d) none of the above.

 (v) Mathematical model of LP problem is important because

(a) it helps in converting the verbal description and numerical data into mathematical expression

(b) decision-makers prefer to work with formal models

(c) it captures the relevant relationship among decision factors

(d) it enables the use of algebraic technique.

 (vi) The initial solution of a transportation problem can be obtained by applying any known method. However, the only condition is that

(a) the solution be optimal (b) the rim conditions are satisfied

(c) the solution not be degenerate (d) all of these.

 (vii) What happens when maximin and minimax values of the game are same?

(a) no solution exists (b) solution is mixed

(c) saddle points exists (d) none of the above.

 (viii) The slack for an activity is equal to

(a) LF - LS (b) EF - ES

(c) LS - ES (d) none of these.

 (ix) If there are n jobs to be performed one at a time on each of m machines, the possible sequences would be

(a) (n!)m (b) (m!)n (c) (n)m (d) (m)n.

 (x) Which of the following relationships is not true?

(a) Ws = Wq + 1/μ (b) Ls = λWs

(c) Ls = Lq + 1/λ (d) Lq = λWq.

**Group – B**

2. (a) A company has two plants, each of which produces and supplies two products: A and B. The plants can each work up to 16 hours a day. In *plant-I*, it takes three hours to prepare and pack 1,000 gallons of A and one hour to prepare and pack one quintal of B. In *plant-II*, it takes two hours to prepare and pack 1,000 gallons of A and 1.5 hours to prepare and pack a quintal of B. In *plant-I*, it costs Rs.15,000 to prepare and pack 1,000 gallons of A and Rs.28,000 to prepare and pack a quintal of B, whereas these costs are Rs.18,000 and Rs.26,000 respectively in *plant-II*. The company is obliged to produce daily at least 10 thousand gallons of A and 8 quintals of B.

 Formulate this problem as a linear programming problem to find out as to how the company should organize its production so that the required amounts of the two products be obtained at minimum cost.

 (b) Solve the following linear programming problem using graphical method and find the optimal solution if exists.

$Minimize Z=4x\_{1}+3x\_{2 }$

$Subject to Constraints:$

$x\_{1}+3x\_{2}\geq 9$

$2x\_{1}+3x\_{2} \geq 12$

$x\_{1}+x\_{2} \geq 5$

$x\_{1},x\_{2}\geq 0$

**7 + 5 = 12**

3. (a) Solve the following LP problem by using Big-M method.

$Minimize Z=4x\_{1}+8x\_{2 }+3x\_{3 }$

$Subject to Constraints:$

$3x\_{1}+2x\_{2}+x\_{3}\leq 3$

$2x\_{1}+x\_{2}+2x\_{3} \geq 3$

$x\_{1},x\_{2}, x\_{3}\geq 0$

 (b) Write down the dual of the following LP problem:

$Minimize Z=2x\_{1}+3x\_{2 }+4x\_{3 }$

$Subject to Constraints:$

$2x\_{1}+3x\_{2}+5x\_{3}\geq 2$

$3x\_{1}+x\_{2}+7x\_{3}=3$

$x\_{1}+4x\_{2}+6x\_{3}\leq 5$

$x\_{1},x\_{2}\geq 0, x\_{3} unrestricted in sign$

 (c) Write down True or False for the following:

(i) An optimal solution does not necessarily use up all the limited resources available.

(ii) If all the constraints are ≥ type inequalities in a LP problem whose objective function is to be maximized, then the solution of the problem is unbounded.

**6 + 4 + 2 = 12**

**Group – C**

4. (a) A company has received a contract to supply gravel to three new construction projects located in towns A, B and C. The construction engineers have estimated that the required amounts of gravel which will be needed at these construction projects are:

|  |  |
| --- | --- |
| Project Location | Weekly Requirement (truckloads) |
| A | 72 |
| B | 102 |
| C | 41 |

The company has 3 gravel pits located in towns X, Y and Z. The gravel required by the construction projects can be supplied by three pits. The amount of gravel that can be supplied by each pit is as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Plant: | X | Y | Z |
| Amount Available (truckloads): | 76 | 82 | 77 |

The company has computed the delivery cost from each pit to each project site. These costs (in Rs) are shown in the following table:

|  |  |  |
| --- | --- | --- |
|  |  | Project Location |
|  |  | A | B | C |
| **Pit** | X | 4 | 8 | 8 |
| Y | 16 | 24 | 16 |
| Z | 8 | 16 | 24 |

(i) Schedule the shipment from each pit to each project in such a manner that it minimizes the total transportation cost within the constraints imposed by pit capacities and project requirements.

(ii) Find the initial solution using Vogel’s Approximation Method (VAM).

(iii) Test for optimality and find the optimal solution.

 (b) How can an unbalanced transportation problem be balanced?

**(2 + 4 + 4) + 2 = 12**

5. (a) A lead draftsman has five drafting tasks to accomplish and five idle draftsmen. Each draftsman is estimated to require the following number of hours for each task.

|  |  |  |
| --- | --- | --- |
|  |  | **Tasks** |
|  |  | A | B | C | D | E |
| Draftsman | 1 | 60 | 50 | 100 | 85 | 95 |
| 2 | 65 | 45 | 100 | 75 | 90 |
| 3 | 70 | 60 | 110 | 97 | 85 |
| 4 | 70 | 55 | 105 | 90 | 95 |
| 5 | 60 | 40 | 120 | 85 | 97 |

If each draftsman costs the company Rs.15.80 per hour, including overhead, find the assignment of draftsmen to tasks that will result in the minimum total cost. What would be the total cost?

 (b) Explain that “assignment problem is a special case of transportation problem.”

**9 + 3 = 12**

**Group – D**

6. (a) Players A and B play a game in which each has three coins, a 5p, l0p and a 20p. Each selects a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, then A wins B's coin. But, if the sum is even, then B wins A's coin. Find the best strategy for each player and the values of the game.

 (b) An insurance company has decided to modernize and refit one of its branch offices. Some of the existing office equipment will be disposed of but the remaining will be returned to the branch after the completion of the renovation work. Tenders are invited from a number of selected contractors. The contractors would be responsible for all the activities in connection with the renovation work excepting the prior removal of the old equipment and its subsequent replacement.

 The major elements of the project have been identified, as follows, along with their durations and immediately preceding elements.

|  |  |  |  |
| --- | --- | --- | --- |
| **Activity** | **Description** | **Predecessor Activity** | **Duration (Days)** |
| A | *Design new premises* | ─ | 14 |
| B | *Obtain tenders from the contractors* | A | 4 |
| C | *Select the contractor* | B | 2 |
| D | *Arrange details with selected contractor* | A | 1 |
| E | *Decide which equipment is to be used* | D | 2 |
| F | *Arrange storage of equipment* | E | 3 |
| G | *Arrange disposal of other equipment* | ─ | 2 |
| H | *Order new equipment* | G | 4 |
| I | *Take delivery of new equipment* | J, H | 3 |
| J | *Renovations take place* | ─ | 12 |
| K | *Remove old equipment for storage or disposal* | A | 4 |
| L | *Cleaning after the contractor has finished* | C, K | 2 |
| M | *Return old equipment for storage* | I, L | 2 |

(i) Draw the network diagram showing the interrelations between the various activities of the project.

(ii) Calculate the minimum time that the renovation can take from the design stage.

(iii) Calculate the 'independent float' that is associated with the non-critical activities in the network diagram.

**3 + (4 + 2 + 3) = 12**

7. Consider the project with activity and duration given as follows:

|  |  |  |
| --- | --- | --- |
| **Activity** | **Immediate predecessor** | **Duration (Days)** |
| A | ─ | 3 |
| B | A | 2 |
| C | A | 6 |
| D | A | 3 |
| E | C, D | 7 |
| F | D | 4 |
| G | E | 3 |
| H | G | 25 |
| I | F, H | 10 |
| J | B, I | 20 |

(i) Draw the network diagram for the project.

(ii) Identify the critical path.

(iii) What is the project duration?

(iv) Find out the total float associated with each activity.

**(4 + 4 +1 + 3) = 12**

**Group – E**

8. (a) A book binder has one printing press, one binding machine and manuscripts of seven different books. The times required for performing printing and binding operations for different books are shown below:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Book : | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Printing time (hours) : | 20 | 90 | 80 | 20 | 120 | 15 | 65 |
| Binding time (hours) : | 25 | 60 | 75 | 30 | 90 | 35 | 50 |

Decide the optimum sequence of processing of books in order to minimize the total time required to bring out all the books.

 (b) A television repairman finds that the time spent on his jobs has an exponential distribution with a mean of 30 minutes. If he repairs the sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution with an approximate average rate of 10 per 8-hour day, what is the repairman's expected idle time each day? How many jobs are ahead of the average set just brought in?

**6 + 6 = 12**

9. (a) Patients arrive at a clinic according to a Poisson distribution at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. The examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic.

(ii) What is the probability that an arriving patient will not wait? Will he find a vacant seat in the room?

(iii) What is the expected waiting time until a patient is discharged from the clinic?

 (b) There are four jobs *A*, *B*, *C* and *D*, which is to be, processed on machines *M*1, *M*2, *M*3 and *M*4 in the order *M*1*M*2*M*3*M*4 .The processing time in hours is given below. Find the optimal sequence as well as idle time of jobs, and waiting time for machines.

|  |  |
| --- | --- |
| *Machine* | *Jobs* |
| *A* | *B* | *C* | *D* |
| *M1* | 15 | 12 | 13 | 16 |
| *M2* | 5 | 2 | 3 | 0 |
| *M3* | 4 | 10 | 6 | 3 |
| *M4* | 14 | 12 | 15 | 19 |

**(3 + 3 + 3) + 3 = 12**