SPECIAL SUPPLE B.TECH/AEIE/BT/CE/CHE/CSE/ECE/EE/IT/ME/1ST SEM/MATH 1101/2018

MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of the determinant $\begin{pmatrix} 100 & 101 & 102 \\ 105 & 106 & 107 \\ 110 & 111 & 112 \end{pmatrix}$ is

(a) 0 (b) 10 (c) 100 (d) 1000

(ii) The equation $x+y+z=0$ has
(a) infinite number of solutions (b) no solution
(c) unique solution (d) two solutions.

(iii) If $y = e^{ax+b}$ then $(y_5)_0 =$
(a) ae^b (b) a^5e^b (c) $a^b e^{ax}$ (d) none of these.

(iv) $\int_0^{\frac{\Pi}{2}} \cos^6 x dx$ is equal to
(a) $\frac{7\Pi}{12}$ (b) $\frac{5\Pi}{32}$ (c) $\frac{\Pi}{32}$ (d) $\frac{3\Pi}{16}$

(v) The series $\sum \frac{1}{n^p}$ is convergent
(a) $p \geq 1$ (b) $p \leq 1$ (c) $p > 1$ (d) $p < 1$

(vi) The sequence $\left\{(-1)^n \frac{1}{n}\right\}$ is

(a) convergent (b) oscillatory
(c) divergent (d) none of these.

(vii) If $u = \frac{x^3 + y^3}{\sqrt{x^2 + y^2}}$, find the value of n so that $xu_x + yu_y = nu$

(a) 0 (b) 2 (c) $\frac{1}{2}$ (d) none of these.

(viii) The value of $\int_C (xdx - dy)$ where C is a line joining $(0,1)$ to $(1,0)$ is

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(ix) Rank of an identity matrix of order 5 is
(a) 0 (b) 5 (c) 25 (d) 1.

(x) The value of $\int_1^0 \int_0^1 (x + y)dxdy =$

(a) 2 (b) 3 (c) -1 (d) 0

Group – B

2\. (a) Without expanding prove that

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

(b) Solve the following system of equations by Cramer's rule

$$3x + y + z = 4$$
$$x - y + 2z = 6$$
$$x + 2y - z = -3$$

6 + 6 = 12

3. (a) If $A=\begin{pmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{pmatrix}$, then show that $A^2-4A-5I=O$, where I,O are the identity matrix and the null matrix of order 3 respectively. Deduce that A is non-singular and hence find A^{-1}.

(b) Find whether the following system is consistent or not by row elimination method

$$x+y+z=1$$
$$2x+y+2z=2$$
$$3x+2y+3z=5$$

6 + 6 = 12

Group – C

4. (a) Verify the Lagrange's Mean Value Theorem for the following function
$f(x)=2x^2-7x+10$, $2\le x\le 5$

(b) Prove that the infinite series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$ converges to 1.

6 + 6 = 12

5. (a) Determine the nature of the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}\cdots$ Justify your answer.

(b) Test the convergence of the series $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\cdots$ to ∞

6 + 6 = 12

Group – D

6. (a) Prove that $y=f(x+ct)+g(x-ct)$ satisfies $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ where f and g are assumed to be at least twice differentiable and c is any constant.

(b) If $u=\sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$

6 + 6 = 12

7. (a) If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$, find $\frac{\partial(u,v)}{\partial(x,y)}$

(b) Find the maxima and minima of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

6 + 6 = 12

Group – E

8. (a) Evaluate $\int_C \{(5xy - 6x^2)dx + (2y - 4x)dy\}$, where C is the arc of the curve $y = x^3$ from the point $(1,1)$ to $(2,8)$ in the xy-plane.

(b) Evaluate $\iint_R (x+y)\,dxdy$ where R is the region in the positive quadrant for which $x + y \le 1$

6 + 6 = 12

9. (a) Find $div\ \vec{F}$ and $curl\ \vec{F}$ where $\vec{F} = grad\,(x^3 + y^3 + z^3 - 3xyz)$

(b) A vector field $\vec{F}$ is given by $\vec{F} = (\sin y)\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral $\int_\tau \vec{F}.\vec{dr}$ where τ is the circular path given by $x^2 + y^2 = a^2$

6 + 6 = 12