(v) Suppose we have a O(n) time algorithm that finds median of an unsorted array. Now consider a QuickSort implementation where we first find median using the above algorithm, then use that median as a pivot. What will be the worst case time complexity of this modified QuickSort?
(a) $O(n^2 \log n)$ (b) $O(n^2)$ (c) O(n logn logn) (d) O(nlogn).

(vi) Solution of the recurrence $T(n) = 2T(n/4) + \sqrt{n}$, T(1) = 1 for n>=0, n is of the form 4^k, k is an integer, is
(a) log n (b) $\sqrt{n}$ log n (c) n (d) n^2.

(vii) In fractional knapsack problem, the best strategy to get the optimal solution, where P_i, W_i is the profit and weight associated with i^{th} object respectively is to
(a) arrange the values P_i/W_i in ascending order
(b) arrange the values P_i in ascending order
(c) arrange the values P_i/W_i in descending order
(d) arrange the values W_i in descending order.

(viii) A problem L is NP-complete if and only if L is NP-hard and
(a) $L \alpha NP$ (b) $L \in NP$ (c) L = NP (d) $L \neq NP$.

(ix) For simultaneously finding the minimum and maximum of n elements the number of comparisons required is at most
(a) $3\lfloor n/2 \rfloor$ (b) 2n (c) 2 (n -1) (d) n log n.

(x) Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is ALWAYS TRUE?
(a) $A(n) = \Omega(W(n))$ (b) $A(n) = \theta(W(n))$
(c) $A(n) = O(W(n))$ (d) $A(n) = o(W(n))$.

Group – B

2\. (a) Write a recursive algorithm to solve Towers of Hanoi problem and then analyze your algorithm in worst case.

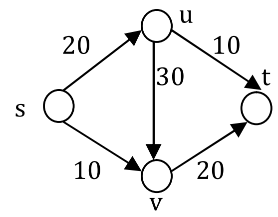
(b) Consider the following input sequence: <5, 2, 4, 7, 1, 3, 2, 6>, on which Merge Sort is applied to arrange them in ascending order. Now answer the following questions:
i) Show all the steps clearly to get the sorted sequence
ii) Analyze the performance of Merge Sort of a set of n numbers in worst case using recursion tree method

(3 + 3) + (3 + 3) = 12

3\. (a) What is the expected number of comparisons required to find a number using sequential search from an array of n numbers where the target number t occurs k times in the array?

(b) What happens if k = 2 in the above question (a), i.e., the target number occurs exactly twice in the above array. Please show the detailed steps of analysis also.

(b) Apply FORD-FULKERSON algorithm on the following flow network to find the maximum flow in the network. s and t denotes source and destination and the weights associated with every edge represents capacity of the respective edge.



(4 × 1.5) + 6 = 12

9\. (a) Define the complexity classes (i) P, (ii) NP, (iii) NP-hard.

(b) Define the term 'polynomial time approximation scheme' in the context of optimization problems.

(c) Give a 2-approximation algorithm for a TSP that satisfies triangle inequality and give a small proof that it indeed maintains the approximation ratio.

3 + 3 + (3 + 3) = 12

M.TECH/CSE/2ND SEM/CSEN 5201/2019

ADVANCED ALGORITHMS
(CSEN 5201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following functions is asymptotically smallest?
(a) 2^n (b) $n^{\log n}$ (c) $n^{\sqrt{n}}$ (d) $\sqrt[3]{\log n}$.

(ii) The augmenting path in a flow network
(a) determines the edge connectivity of a network
(b) comprises edges which can admit positive flow
(c) converts a tree network into a cyclic chain
(d) is used only for cyclic chain networks

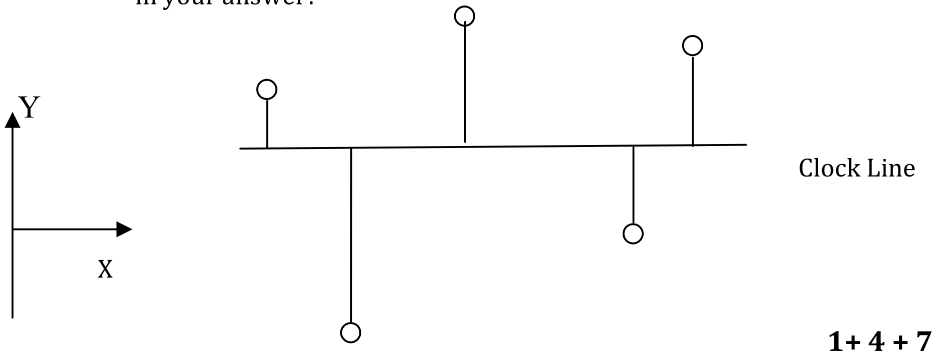
(iii) Consider the following code snippet which finds greatest common divisor (gcd) of two integers n and m, where n ≥ m:

```
int gcd(n, m)
{
  if (n%m ==0) return m;
  n = n%m;
  return gcd(m, n);
}
```

How many recursive calls are made by the above gcd function?
(a) θ(logn) (b) Ω(n) (c) θ(loglogn) (d) θ(sqrt(n)).

(iv) The single-source shortest path problem in a graph G becomes undefined if there exists ________________________from the source.
(a) a –ve weight cycle in the graph
(b) a –ve edge reachable from the source
(c) multiple –ve edges in the graph
(d) a –ve weight cycle reachable from the source.

(c) Mr. Hero is a VLSI engineer. He wants to connect n circuit points to the clock signal. Now, the clock signal is going to pass parallel to the x-axis and all those circuit points are going to be connected by wires which are all vertical to the clock line. Please look at the adjoining figure to have a feel. Now, if the coordinates (x_i, y_i) for each circuit point c_i to be connected are given, how will you determine the optimal placement of the clock line so that the total wire-length L for connecting the circuit points to the clock line is minimized. In other words, if the equation of the clock line is $y = y_c$ then what should be the value of y_c in order to minimize L = $\sum_{i=1}^{n}(|y_i - y_c|)$ in your answer?

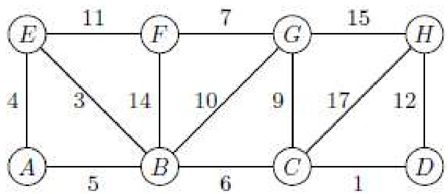


1+ 4 + 7 = 12

Group – C

4 .(a) Define a spanning tree of a graph.

Consider the weighted graph below. Apply priority queue based Prim's algorithm on this graph starting from vertex A. Write the edges in which order they are added to the minimum spanning tree (MST).



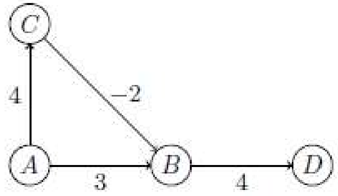
(b) Let (u, v) be a minimum weight edge in an undirected graph G. Show that (u, v) belongs to some minimum spanning tree of G.

Can you explain why in the above statement we used the word 'some' and not 'every'? Also in which special cases the word 'every' can be used in place of 'some'?

(1+ 6)+(3 + 2) = 12

5. (a) Describe the Bellman Ford algorithm for finding shortest paths using a suitable example.

(b) What will happen when you are going to apply Dijkstra's algorithms on the following graph, using vertex A as the source? Give justification against your answer.



7 + 5 = 12

Group – D

6. (a) Matrices A, B and C, of dimensions 10 × 25, 25 × 6 and 6 × 15 respectively, are provided. You are told to ensure that when computing the matrix product A.B.C the number of scalar multiplications will be a minimum. How would you parenthesize the product, and what would be the minimum number of scalar multiplications you would need to perform?

(b) Extend the above idea to find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <30, 35, 15, 5, 10, 20, 25> using dynamic programming method.

4 + 8 = 12

7. (a) A sequence of n operations is performed on a data structure. The cost of i^{th} operation is $C(i) = i^2$, if i is an exact power of 3
= 3, otherwise.
Calculate the exact expression for finding the cost for n successive operations using aggregate analysis. Determine the asymptotic amortized cost per operation. To make things simple, let us assume that n is an exact power of 3.

(b) Give the pseudo-code for Euclid's GCD algorithm and illustrate it with one small example.

8 + 4 = 12

Group – E

8. (a) State the following problems and also give a one-line comment on their hardness.
(i) Eulerian Path Problem
(ii) Hamiltonian Path Problem
(iii) Edge Cover problem
(iv) Vertex Cover Problem