B.TECH/ME/4TH SEM/MATH 2001/2019

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of $\oint_C \frac{dz}{z-4}$ where $C:|z-1|=2$ is

(a) $2\pi i$ (b) 0 (c) πi (d) $4\pi i$

(ii) Which of the following functions has essential singularity at $z=0$?

(a) $\frac{1}{z}$ (b) $\frac{1}{z}+\frac{1}{z^2}$ (c) e^{z^2} (d) $e^{\frac{1}{z^3}}$

(iii) $\int_{-1}^{1} P_m(x)P_n(x)dx$ where $n \neq m$ is

(a) 1 (b) -1 (c) 0 (d) 2

(iv) The inverse Fourier transform of $f(s)=\frac{1}{s}$ is

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{2}{\pi}$ (d) $\frac{\pi^2}{4}$

(v) If the Fourier transform of $f(t)$ is $F(s)$, then the Fourier transform of $f(-3t)$ is

(a) $\frac{1}{3}F\left(\frac{3}{s}\right)$ (b) $-\frac{1}{3}F\left(\frac{s}{3}\right)$ (c) $\frac{1}{3}F\left(\frac{s}{3}\right)$ (d) $\frac{1}{3}F\left(\frac{-s}{3}\right)$

(vi) The value of $P_3(x)$ is

(a) $\frac{1}{2}(5x^3-3x)$ (b) $\frac{1}{2}(5x^3+3x)$ (c) $\frac{1}{3}(5x^3-3x)$ (d) $\frac{1}{2}(5x^3-3x^2)$

(vii) In the Fourier series of $f(x)=\frac{x(\pi^2-x^2)}{124}$ in $(-\pi,\pi)$, the value of a_0 is

(a) $\frac{\pi^2}{124}$ (b) -1 (c) 1 (d) 0

(viii) Particular Integral (P.I.) of $(5D^2-DD'+6D'^2)z=\sin(x-3y)$ is

(a) $\frac{\sin(x-3y)}{-56}$ (b) $\frac{\sin(x-3y)}{56}$

(c) $\frac{\sin(3x-y)}{-56}$ (d) $\frac{\sin(x-3y)}{62}$

(ix) The complementary function of $r+t+2s=0$ is

(a) $z=\phi_1(y-x)+x\phi_2(y-x)$ (b) $z=\phi_1(y-x)+\phi_2(y+2x)$

(c) $z=\phi_1(y-x)+\phi_2(y+x)$ (d) $z=\phi_1(y-x)+x\phi_2(y+x)$

(x) The value of $J_{-n}(x)$ is

(a) $(-1)^{-n}J_n(x)$ (b) $(-1)^n J_n(x)$

(c) $(-1)^{n+1}J_n(x)$ (d) $(-1)^n J_{-n}(x)$

Group – B

2. (a) Show that $Lt_{z\to 0}\frac{\bar{z}}{z}$ does not exist.

(b) Find the values of a and b such that the function $f(z)=x^2+ay^2-2xy+i(bx^2-y^2+2xy)$ is analytic.

(c) Show that $f(z)=\begin{cases}\frac{xy}{x^2+y^2}, & z\neq 0\\ 0, & z=0\end{cases}$, is continuous at $z=0$.

4 + 4 + 4 = 12

3. (a) Evaluate $\oint_C \frac{3z^2+z+1}{(z+3)(z^2-1)}dz$ by Cauchy's integral formula where $C:|z-i|=2$.

(b) Find the nature of the singularities of the following functions:

(i) $f(z)=\frac{\sin z - z}{z^3}$ (ii) $f(z)=\operatorname{cosec}\left(\frac{\pi}{z}\right)$.

6 + 6 = 12

Group – C

4\. (a) Find the Fourier series of the following half-wave rectifier function $f(x)=\begin{cases} 0, -\pi \le x \le 0 \\ \sin x, \quad 0 \le x \le \pi \end{cases}$

(b) Find $F_S\{2e^{-3t}\cos 4t\}$.

7 + 5 = 12

5\. (a) Find the Fourier sine series of $f(x)=x+1$ in $0<x<\pi$. Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots=\frac{\pi}{4}$.

(b) Find the inverse Fourier transform of $f(s)=\dfrac{1}{s^2+4s+13}$.

7 + 5 = 12

Group – D

6\. (a) Find the series solution of $(1-x^2)y''-2xy'+2y=0$ about $x=0$.

(b) Show that $J_{\frac{1}{2}}(x)=\sqrt{\dfrac{2}{\pi x}}\sin x$.

7 + 5 = 12

7\. (a) Use finite difference method to find the solution of the following boundary value problem: $xy''+3y'+(1+x)y=1+x^2, \quad y(0)=1, y(4)=0$ at $x=1, 2, 3$.

(b) Prove that $nP_n(x)=xP_n'(x)-P_{n-1}'(x)$.

6 + 6 = 12

Group – E

8\. (a) Solve the following partial differential equation by Lagrange's method: $(y-z)p+(x-y)q=z-x$.

(b) Solve the following non-linear partial differential equation by Charpit's method: $px+qy=pq$.

6 + 6 = 12

9\. (a) Solve the following homogenous linear partial differential equation: $\dfrac{\partial^2 z}{\partial x^2}-2\dfrac{\partial^2 z}{\partial x\partial y}+\dfrac{\partial^2 z}{\partial y^2}=e^{x+2y}$.

(b) Solve the following partial differential equation by the method of separation of variables:
$\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial t}+u,$ where $u(x,0)=6e^{-3x}$ and u is a function of x and t.

5 + 7 = 12