B.TECH/EE/6TH SEM/ ELEC 3231/2019
DIGITAL SIGNAL PROCESSING
(ELEC 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The common period (N) of the signal $x(n) = \cos(0.4\pi n) + \sin(0.5\pi n + 30^0)$ is
(a) N = 20 (b) N = 5 (c) N = 4 (d) N = 2.

(ii) From a given signal x(n), a signal $x(-n-\alpha)$ can be generated using one of the following sequence of operations.
(a) time reversal of $x(n)$ and then delayed by α
(b) $x(n)$ delayed by α and then time reversal
(c) $x(n)$ advanced by α and then time reversal
(d) time reversal of $x(n)$ and then upscaling by α.

(iii) The value of y(2) using linear convolution $(y(n) = x(n) * h(n))$ of the sequences $x(n) = (3,\ 2,\ 4), 0 \le n \le 2$ and $h(n) = (-2,\ 3),\ 0 \le n \le 1$ is
(a)12 (b) 5 (c) -2 (d) -6.

(iv) The z –transform of a 'left sided' signal $X(z) = \frac{z}{z-b}$ convergences if ROC is
(a) inside an annulus bounded by $|b| < |z| < \infty$
(b) outside a circle of radius $|z| > |b|$
(c) on a circle of radius $|z| = |b|$
(d) inside a circle of radius $|z| < |b|$.

(v) If the z – transform of a signal (x(n)) is $X(z) = \frac{z}{z-0.5}$; ROC: $|z| > 0.5$, then z – transform of $\bar{x}(n) = nx(n)$ is
(a) $\bar{X}(z) = \frac{1}{z(z-0.5)}$; ROC: $|z| < 0.5$
(b) $\bar{X}(z) = \frac{(z-1)}{(z-0.5)}$; ROC: $0.5 < |z| < 1$
(c) $\bar{X}(z) = \frac{(z-0.5)}{1}$; ROC: $|z| > 0.5$
(d) $\bar{X}(z) = \frac{0.5z}{(z-0.5)^2}$; ROC: $|z| > 0.5$.

(vi) A periodic signal x(n) is expressed as $x(n) = 1 + 0.666\cos\left(\frac{2\pi}{3}n + 120^0\right)$, the corresponding normalized power signal P_x is
(a)1.666 (b) 1.444 (c) 1.105 (d) 1.222.

(vii) A periodic signal x(n) (with fundamental period N = 3) is expressed in exponential form $x(n) = \sum_{k=0}^{2} D_k e^{j\left(\frac{2\pi}{3}\right)kn}$ where, $D_0 = 1,\ D_1 = 0.333\angle 120°$, and $D_2 = 0.333\angle -120°$. The time domain signal x(n) can be expressed from DTFT coefficients as
(a) $x(n) = 1 + 0.333\sin\left(\frac{\pi n}{3} + 120\right)$
(b) $x(n) = 0.707 + 0.235\cos\left(\frac{2\pi n}{3} - 120\right)$
(c) $x(n) = 1 + 0.666\cos\left(\frac{2\pi}{3}n + 120\right)$
(d) $x(n) = 1.33 + 0.666\sin\left(\frac{2\pi n}{3} - 120\right)$.

(viii) The DFT coefficient X(2) of the four point segment $x(0) = 0,\ x(1) = 1,\ x(2) = 2,\ x(3) = 3$ of a sequence x(n) is
(a) X(2) = 6 (b) X(2) = −2 (c) X(2) = −2 + j2 (d) X(2) = −2 − j2.

(ix) The equivalent digital filter H(z) for the given analog filter $H_c(s) = \frac{1}{s+1}$ using impulse invariance mapping with sampling time $T_s = 1s$ is
(a) $H(z) = \frac{z}{z-0.368}$ (b) $H(z) = \frac{z-1}{z-0.368}$
(c) $H(z) = \frac{z}{z-0.638}$ (d) $H(z) = \frac{z}{z-1}$.

(x) If a causal and stable discrete time system $H(z) = \frac{z}{z-0.8}$ is excited with a sinusoidal input $x(n) = \cos(0.05\pi n)\,u(n)$ then the amplitude (B) of the sinusoidal output response $y_{ss}(n) = B\cos(0.05\pi n + \theta)$ at steady-state is
(a) 6 (b) 4.1 (c) 4.5 (d) 2.5

Group – B

2. (a) A non-recursive filter of length (M + 1) is described by $y(n) = b_0x(n) + b_1x(n-1) + b_2x(n-2) + \cdots \ldots + b_Mx(n-M)$. Show that the impulse response sequence or samples are the filter 'tap-coefficients' (i.e. $b_i = 0,\ 1,\ 2, \ldots \ldots . M$).

(b) (i) If x(n) is the input and h(n) is the impulse response of a linear time-invariant discrete system 'S', then the show that the output y(n) is given by $y(n) = \sum_{m=0}^{\infty} x(m)h(n-m)$(assume both the signals x(n), & $h(n)$ are causal).

(ii) Convert a non-recursive filter $y(n) = x(n) + x(n-1) + x(n-2)$ to an equivalent recursive form and hence compute recursively the values of y(n) for n = 0 to 5 with $x(n) = \delta(n)$ and $y(-1) = 0$. Comment on the length of the output sequence.

3 + (5 + 4)=12

3. (a) Prove that linear time invariant system with impulse response h(n)is stable, in the bounded-input bounded-output sense, if and only if the impulse response is absolutely summable, that is, if $\sum_{n=-\infty}^{\infty}|h(n)| < \infty$.

(b) Realize the following digital filter using "direct-form-II" structure. $y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$. Is the digital filter stable?

5 + 7 =12

Group – C

4. (a) Let $X(z) = Z(x(n))$ with ROC: R_x, if the signal $x(n), n \geq 0$ is multiplied by n, then show that $Z(nx(n)) = -z\frac{dX(z)}{dz}$, with ROC: R_x.

(b) Find the inverse Z-transform of $H(z) = \frac{1}{(1-z^{-1})(1-0.5z^{-1})}$, and its ROC. Assume h(n) is left–sided signal.

5 + 7 = 12

5. (a) (i) What is the basic principle behind the impulse-invariance transformation?

(ii) How does the $s-plane$ get mapped into the $z-plane$ under the impulse-invariance transformation?—Discuss.

(b) Transform a filter $H(s) = \frac{s+1}{s^2+5s+6}$, into an equivalent digital filter H(z) using the impulse invariance technique in which $T_s = 0.1s$ and hence express H(z) as an equivalent difference equation for computer simulation.

(2 + 3) +7=12

Group – D

6. (a) Define the DFT of a finite sequence. Show that the N − point DFT of a finite sequence signal x(n) is periodic with period 'N'.

(b) Given a sequence $x(n) = \{1, 2, 1, 0\}$ for $0 \leq n \leq 3$, evaluate DFT of a shifted signal $y(n) = x(n-1)$, i.e. Y(k). Assume that $f_s = 100$ Hz. Sketch the amplitude spectrum, phase spectrum and power spectrum in context with the signal y(n).

4 + 8 = 12

7. (a) (i) What is circular convolution, why should we care about it, and how is it different from the linear convolution?

(ii) If $x(n) = \{6, 2, a, 0, b\}$, for $n = 0, 1, 2, 3, 4$ is circularly even symmetric, find the values of 'a' and 'b'.

(b) Let $x(n) = \{1, -1, -1, 1\}$ for $n = 0, 1, 2, 3$, and $h(n) = \{1, 2, 2, 1\}$, for $n = 0, 1, 2, 3$. Determine the output response y(n) using-circular convolution $(x(n) \otimes h(n))$.

(3 + 3) + 6=12

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Group – E

8. (a) (i) Show that any rational system H(z) can be decomposed into a minimum-phase system and an all-pass system.

(ii) It is said that IIR filters cannot have linear phase. Do you agree or disagree? Explain.

(b) The impulse response of a FIR filter is given as $h(n) = \{2, 1, 0, -1, -2\}$, for $n = 0, 1, 2, 3, 4$. Is it a linear-phase filter? If so, what type? Obtain the transfer function of the FIR filter (system) and sketch the pole-zero distributions of the FIR filter.

(2 +3) + 7 = 12

9. (a) A lowpass FIR filter having the following frequency specifications: Passband edge frequency $\Omega_p = 0.375\pi$ rad/s; stopband edge frequency, $\Omega_s = 0.5\pi$ rad/s; cut-off frequency, $\Omega_c = 0.438\pi$ rad/s; transition band, (width)=0.125π rad/s; passband ripple, $\delta_p \leq 0.0575$, stopband, $\delta_s \leq 0.0032$. Sketch the tolerance diagram for the low pass-filter indicating all design specifications.

(b) Design a low-pass FIR filter with the specifications given in Q.9 (a) using the "Hamming Window" technique.

4 + 8 =12