(vii) The joint density function of X, Y is given by :

$$f(x,y) = \begin{cases} k(x+y), & 0<x<1,\ 0<y<1 \\ 0, & \text{otherwise} \end{cases}$$

The value of k is

(a) 1 (b) 2 (c) 0.5 (d) 3.

(viii) If $\rho(U,V) = 0.5$, $U = -5X + 0.9$, $V = 2Y$, then $\rho(X,Y) =$

(a) -0.5 (b) 0.5 (c) 1 (d) 0.

(ix) If X is a random variable, then [where a, b are constants]

(a) $E(aX+b)=a^2E(X)$ (b) $E(aX+b)=(a+b)E(X)$

(c) $E(aX+b)=aE(X)+b$ (d) $Var(aX+b)=aVar(X)$.

(x) Consider a Markov chain with state space $S=\{0,1\}$ and probability transition matrix $P=\begin{pmatrix} a & 1-a \\ 1-b & b \end{pmatrix}$. If Initially it was at the state "0", then the probability to stay at the state "1" after one step is

(a) b (b) $1-b$ (c) a (d) $1-a$.

Group – B

2. (a) Solve the following equations by LU-factorization method

$$2x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

(b) Find the real root of the equation $\log x - \cos x = 0$, which lies in $(1,2)$, correct to three places of decimal by Newton-Raphson's method.

7 + 5 = 12

3. (a) A test performed on NPN transistor gives the following result:

Base Current $f(mA)$	0	0.01	0.02	0.03	0.04	0.05
Collector current $I_c(mA)$	0	1.2	2.5	3.6	4.3	5.34

Use Newton's backward interpolation formula to evaluate the value of collector current for the base current of $0.045mA$.

(b) Given $\frac{dy}{dx}=-xy^2$ with the initial condition $y(2)=1$. Use modified Euler's method to compute the value of y for x=2.1, taking h = 0.1 (perform only two iterations).

6 + 6 = 12

Group – C

4. (a) If you flip a fair coin three times, determine the probability of the events given below;
   (i) three heads: HHH,
   (ii) the sequence head, tail, head: HTH,
   (iii) any sequence with 2 heads and 1 tail,
   (iv) any sequence where the number of heads is greater than or equal to the number of tails.

(b) Alice and Bob each chose at random a number in the interval [0,2]. We assume a uniform probability law under which the probability of an event is proportional to its length. What is the probability that at least one of the numbers is greater than $\frac{1}{3}$.

(2+2+3+3) + 2 = 12

5. (a) You have a fair five-sided die. The sides of the die are numbered from 1 to 5. Each die roll is independent of all others and all faces are equally likely to come out on top when the die is rolled. Suppose you roll the die twice.

Let event A to be "the total of two rolls is10 ", event B be "at least one roll resulted in 5", and event C be "at least one roll resulted in 1".

(i) Is event A independent of event B? Justify.
(ii) Is event A independent of event C? Justify.

(b) You go to the shop to buy a toothbrush. The toothbrushes there are red, blue, green, purple and white. The probability that you buy a red toothbrush is three times the probability that you buy a green one; the probability that you buy a blue one is twice the probability that you buy a green one; the probabilities of buying green, purple, and white are all equal. You are certain to buy exactly one toothbrush. For each colour, find the probability that you buy a toothbrush of that colour.

(3 + 3) + 6 = 12

Group – D

6. (a) An archer shoots an arrow at a target. The distance of the arrow from the centre of the target is a random variable X whose p.d.f. is given by :

$$f(x) = \frac{(3+2x-x^2)}{9}, x \le 3$$
$$= 0, \text{ otherwise}$$

The archer's score is determined as follows:

Distance	X<0.5	0.5≤ X≤1	1≤X≤2	2≤X≤3	X≥3
Score	10	7	4	1	0

Construct the probability mass function for the archer's score, and find the archer's expected score.

(b) What is the probability that a binary string (string consisting of 0's and 1's) of a finite length n , will have an even numbers of 0 s ?[Assume n is even]

7 + 5=12

7. (a) Suppose that the number of typographical errors on a single page of a book has a Poisson distribution with parameters $\lambda = 1$. Calculate the probability that there is at least one error in this page.

(b) Let c be a constant. Show that

(i) $\mathrm{Var}(cX) = c^2\mathrm{Var}(X)$

(ii) $\mathrm{Var}(c+X) = \mathrm{Var}(X)$

(c) The power W dissipated in a resistor is proportional to the square of the voltage V, i.e., $W\ rV^2$, where r is a constant. If $r = 3$ and V be assumed to be a normal random variable with mean 6 and standard deviation 1, find $P(W>120)$ [Voltage V is assumed to be always nonnegative].

3 + 4 + 5= 12

Group – E

8. (a) Three balls *a,b,c* are randomly distributed into three boxes (a box may contain any number of balls). Let X_1 and X_2 be the number of balls in box 1 and box 2 respectively. Find the joint p.m.f. of X_1 and X_2. Also compute $E(X_1+X_2)$ and the marginal p.m.f of X_1 and X_2.

(b) The joint p.d.f. of X and Y is $f(x,y)=\begin{cases} 8xy, & 0<x<y<1 \\ 0 & ,\text{elsewhere} \end{cases}$.

Find the marginal p.d.f. of X and Y. Also find the conditional p.d.f. $f_{X|Y}(x|y)$. Are X and Y independent?

6 + 6 = 12

9. (a) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged . Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps ? In the long run , what is the probability that there are 2 red marbles in the urn A ?

(b) (i) State Chapman-Kolmogorov equation.

(ii) Let $\{X_n, n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix

$$\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

and the initial distribution $P(X_0 = i) = \frac{1}{3}, i = 0,1,2$.

Find $P(X_2 = 2, X_1 = 1, X_0 = 2)$

8 + 2 + 2 =12

B.TECH/ CSE /4TH SEM/ MATH 2202/2019
PROBABILITY & NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In Simpson's $\frac{1}{3}$ rule the function is approximated by a

(a) circle (b) ellipse (c) parabola (d) line.

(ii) The rate of convergence of Regula-Falsi method is

(a) linear (b) quadratic (c) cubic (d) none of these.

(iii) If $y(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n, a_n \neq 0$ then $\Delta^n y(x)$ is, (where h is the step length)

(a) 0 (b) $a_nh^nn!$ (c) 1 (d) $a_0h^nn!$.

(iv) Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$ (c) $\frac{8}{15}$ (d) $\frac{9}{20}$.

(v) Pulse rates of adult men are approximately normal with a mean of 70 and a standard deviation of 8. Which choice correctly describes how to find the proportion of men that have a pulse rate greater than 78?

(a) Find the area to the left of z =1 under a standard normal curve.
(b) Find the area between z=–1 and z =1 under a standard normal curve.
(c) Find the area to the right z = 1 under a standard normal curve
(d) Find the area to the left of z= –1 under a standard normal curve.

(vi) The value of $\int_1^5 x^2\,dx$ using trapezoidal rule is: [assume that there are 4 equal subintervals]

(a) 40 (b) 41 (c) 42 (d) 43.