

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: 10 × 1 = 10
- (i) The Hamiltonian of a system is a function of
(a) q_j, p_j, t (b) q_j, \dot{q}_j, t (c) q_j, \dot{p}_j, t (d) None of (a),(b),(c).
- (ii) The degrees of freedom relation for a system of N particles with K constraints is given by
(a) $N - K$ (b) $3N - K$ (c) $N - 3K$ (d) $3K - N$.
- (iii) Planck's constant has the dimensions of
(a) Energy (b) Force
(c) Linear momentum (d) Angular momentum.
- (iv) The correct commutator relations are:
(a) $[\hat{x}, \hat{p}_x] = i\hbar$ (b) $[\hat{x}, \hat{p}_y] = 0$
(c) $[\hat{y}, \hat{p}_y] = 0$ (d) Both (a) and (b).
- (v) The spacing between the nth energy states and the next energy state in a one dimensional potential box increases by
(a) $(2n - 1)$ (b) $(2n + 1)$ (c) $(n - 1)$ (d) $(n + 1)$.
- (vi) The relation between entropy (S) and thermodynamical probability (W) is given by
(a) $S = \ln W$ (b) $S = K_B \ln W$ (c) $S = e^W$ (d) $S = e^{\ln W}$.
- (vii) The electric displacement, \vec{D} is expressed through the following relationship between the polarization vector \vec{P} , the electric field \vec{E} , and the permittivity of free space ϵ_0
(a) $\vec{D} = \epsilon_0 \vec{E} - \vec{P}$ (b) $\vec{D} = \vec{P} - \epsilon_0 \vec{E}$
(c) $\vec{D} - \epsilon_0 \vec{E} = \vec{P}$ (d) none of (a),(b),(c).

- (viii) In an intrinsic semiconductor, the acceptor level
(a) lies near the valence band edge
(b) lies near the conduction band edge
(c) lies halfway between the valence and conduction band edges
(d) does not exist
- (ix) If χ_m is the magnetic susceptibility of a ferromagnetic material above its transition temperature T_c , the Curie-Weiss law is given by
(a) $\chi_m = C(T - T_c)$ (b) $\chi_m = \frac{C}{T - T_c}$
(c) $\chi_m = \frac{T - T_c}{C}$ (d) $\chi_m = \frac{C}{T + T_c}$
- (x) The group velocity of electron from E-k curve is
(a) $v = \frac{1}{\hbar} \frac{dE}{dk}$ (b) $v = \hbar \frac{dE}{dk}$ (c) $v = \frac{1}{\hbar} \frac{d^2E}{dk^2}$ (d) $v = \frac{\hbar}{\left(\frac{dE}{dk}\right)}$.

Group – B

2. (a) Two masses m_1 and m_2 ($m_1 \neq m_2$) are attached at two ends of a massless string. The string is mounted over a massless pulley.
i) Write down the constraint relations and classify them.
ii) Construct the Lagrangian of the system
iii) Find the equation of motion.
- (b) Show that the Hamiltonian remains conserved for a system where Lagrangian does not explicitly depend on time.
- (c) The Lagrangian of a system is given by $L = \frac{1}{2} \alpha \dot{q} - \frac{1}{2} \beta q^2$, where α and β are the constants. Obtain the Hamiltonian of the system.
((2 + 1) + 2 + 2) + 3 + 2 = 12
3. (a) Write down the time-dependent Schrodinger equation in one dimension, and define all the terms. When can this equation be reduced to the time-independent Schrodinger equation? What is the physical interpretation of the wave function?
- (b) Evaluate the commutator $[\hat{x}, \hat{p}_x^2]$.
- (c) If $\hat{\alpha}, \hat{\beta}$, and $\hat{\gamma}$ are three linear operators, then prove that
$$[\hat{\alpha}, [\hat{\beta}, \hat{\gamma}]] + [\hat{\beta}, [\hat{\gamma}, \hat{\alpha}]] + [\hat{\gamma}, [\hat{\alpha}, \hat{\beta}]] = 0$$
- (d) Calculate the expected average of P_x for the wave function
$$\varphi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

in the region from $x = 0$ to $x = L$.
(2 + 1 + 1) + 2 + 3 + 3 = 12

Group – C

4. (a) Compare Maxwell-Boltzmann (MB), Fermi-Dirac (FD), and Bose-Einstein (BE) statistics mentioning at least three characteristics.
- (b) Two particles are distributed into three energy levels having energies E, 2E, and 3E so that the total energy is 4E. If the levels are degenerate with degeneracy 2, 1, 1 respectively, write down the macrostates and calculate the corresponding number of microstates for Maxwell-Boltzmann particles, Fermi-Dirac particles, and Bose-Einstein particles.
- (c) Using Fermi-Dirac statistics, calculate the concentration of electrons in the conduction band of an intrinsic semiconductor.

$$2 + (2 + 2 + 2) + 4 = 12$$

5. (a) Write down the Bose-Einstein distribution function. Develop an expression for the density of states of a photon gas in equilibrium. Hence derive Planck's radiation law for black bodies.
- (b) Derive an expression for the Fermi level energy (at $T = 0$ K) in metals as a function of the electron concentration starting from Fermi-Dirac distribution.
- (c) The density of zinc ($1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2$) is 7130 kg-m and its atomic weight is 65.4. Calculate its Fermi energy at $T = 0$ K.
- (d) Calculate the Fermi temperature of silver (Fermi energy of silver is 5.6 eV, Avogadro number = 6.023×10^{23} , $k = 8.6 \times 10^{-5}$ eV/K).

$$(1 + 2 + 3) + 3 + 2 + 1 = 12$$

Group – D

6. (a) Write down the expressions for the electric field between the plates of a parallel plate capacitor i) without dielectric, and ii) with a dielectric between the plates. Using these, derive the expressions for the capacitance of a parallel plate capacitor in both the cases.
- (b) Derive an expression for electric polarization of a dielectric medium.
- (c) If the susceptibility of a gas of CO₂ molecules is 0.985×10^{-3} , calculate the electric polarizability of CO₂. The density of the gas is 1.977 kg/m³. (Molecular weight of CO₂ is 44 grams/ moles, Avogadro number = 6.023×10^{23}).

$$(2 + 3) + 4 + 3 = 12$$

7. (a) Consider a rectangular current carrying loop placed in a uniform magnetic field such that the magnetic field is always perpendicular to two of the loop's parallel edges. Derive an expression for the torque on this current carrying loop due to the magnetic field. Use this result to define the magnetic dipole moment of this current carrying loop.
- (b) In hydrogen atom, an electron revolves around a nucleus in an orbit of 0.50×10^{-10} m radius. If the frequency of revolution of an electron is 6.0×10^{15} Hz, find the magnetic moment of the orbiting electron and express the value in term of Bohr magneton.
- (c) State Weiss' postulates for ferromagnetic materials and hence derive the Curie-Weiss law.

$$(4+1) + (2+2) + (1+2) = 12$$

Group – E

8. (a) State Bloch's theorem in one dimension. What are the basic features of the Kronig-Penney model? Mention at least two points.
- (b) The band energy of an electron is given by $E = \alpha + \beta k^2$, where α and β are constants. Find the group velocity and the effective mass of the electron.
- (c) An electron is moving in one dimension in a region where the potential is given by

$$V(x) = V_0 [1 + \cos(2\pi x/a)],$$

Where V_0 and a are constants. Show that if $\psi(x)$ is the wave function of the electron, then its wave function at $x + a$ can be written in the form

$$\psi(x + a) = e^{i\theta} \psi(x).$$

$$(2+2) + 4 + 4 = 12$$

9. (a) What is the critical magnetic field for a superconductor? How does it vary with temperature?
- (b) Lead (Pb) gets transition to its superconducting state at 7.20 Kelvin. Lead has critical magnetic field at 0K is 65100 A/m, calculate its critical magnetic field at -271°C .
- (c) Distinguish between types I and type II superconductors. Name some materials belonging to these two types of superconductors.
- (d) Write down the London equation of superconductivity in terms of magnetic field induction and hence describe Meissner effect.

$$(2 + 1) + 2 + (2 + 1) + (1 + 3) = 12$$