B.TECH/ BT/ CE/ EE/ ME /4TH SEM/ MATH 2002/2019
NUMERICAL AND STATISTICAL METHODS
(MATH 2002)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) Taking h = 1, $\Delta^2(e^x)$ is equal to
        (a) $(e-1)e^x$ (b) $(e-1)^2 e^x$ (c) $(e-1)^2 e^{2x}$ (d) e^{2x}.

   (ii) If $\frac{dy}{dx} = x^2 + y^2, y(1) = 0$ then the value of y(1.1) using Euler's method by taking h = 0.1 is
         (a) 0.2 (b) 0.3 (c) 0.1 (d) 0.5.

   (iii) In the Simpson's one third rule, the area bounded by the curve Y = f(X), X –axis, X = X_0 and X = X_n is approximated by the area of
          (a) rectangle (b) parabola (c) ellipse (d) straight line.

   (iv) The order of convergence of Newton-Raphson method is
         (a) 0 (b) 1 (c) 2 (d) 3.

   (v) A matrix *A* can be factorized into lower and upper triangular matrix if all the principal minors of *A* are
        (a) singular (b) non-singular (c) zero (d) have determinant value ±1.

   (vi) If X is a continuous random variable with p.d.f. $f(x) = e^{-x^2}, -\infty < x < \infty$, then $E(100X^3 + 99X)$ is
         (a) 0 (b) 2 (c) −2 (d) 1.

   (vii) The mean and variance of a Binomial distribution B(n, p) are 10 and $\frac{10}{3}$ respectively. Then the value of n and p are
          (a) $\left(15, \frac{2}{3}\right)$ (b) $\left(15, \frac{1}{3}\right)$ (c) $\left(10, \frac{2}{3}\right)$ (d) $\left(10, \frac{1}{3}\right)$.

   (viii) Suppose U and V are two random variables with correlation coefficient $r_{UV} = -0.5$. If $U = -2.99X + 100$ and $V = 5Y - 20$ then r_{XY} is
           (a) −0.5 (b) 0.5 (c) 0 (d) 1.

   (ix) If X and Y are independent of each other, then $r(X, Y)$ is
         (a) −1 (b) 0 (c) 1 (d) 0.5.

   (x) If A and B are two events with P(A)=0.4, P(B)=0.3 and P(A ∩ B) = 0.2, then $P(A^c \cap B)$ is
        (a) 0.2 (b) 0.3 (c) 0.1 (d) 0.5

Group – B

2. (a) Using bisection method find a real positive root of the equation $\sin x = \frac{1}{x}$ correct to two places of decimal.

   (b) Solve the following system of equations by Gauss-elimination method:
        $$x + 3y + 2z = 5$$
        $$2x - y + z = -1$$
        $$x + 2y + 3z = -2$$
        6 + 6 = 12

3. (a) Solve the following system of equations by Gauss-Seidel's iteration method, using four iterations:
        $$10x + 2y + z = 9$$
        $$x + 10y - z = -22$$
        $$-2x + 3y + 10z = 22$$

   (b) Compute, by the Newton-Raphson method, the root of the equation $x^3 - x - 1 = 0$ that lies between x = 1 and x = 2, correct to three places of decimal.
        7 + 5 = 12

Group – C

4. (a) Calculate f(5) by using Lagrange's interpolation formula, given the following data

x	3	7	9	12
$f(x)$	11	17	24	30

   (b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{1 - 0.162 \sin^2 x}\, dx$, taking five equal intervals. Explain the reason behind your choice of the integration formula used.
        6 + 6 = 12

5. (a) Evaluate y(1.1) and y(1.2) using Runge-Kutta method of order 4 for the initial value problem $\frac{dy}{dx} = x^2 + y^2, y(1) = 0$, taking h=0.1.

   (b) The population of a town in decennial census were given in the following table.

Year	1921	1931	1941	1951	1961
Population (in thousands)	46	66	81	93	101

   Estimate the population for the year 1955.
        6 + 6 = 12

Group – D

6. (a) A man alternatively tosses a coin and throws a die beginning with coin. What is the probability that he will get a head before he gets a 5 or 6 on the die?

(b) A bag contains 5 white and 4 black balls. If 3 balls are drawn at random, what are the probabilities of the following?
(i) two of them are white
(ii) at most one of them is white
(iii) at least two are white.

6 + 6 = 12

7. (a) There are two identical urns containing 4 white and 3 red balls; 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. What is the probability that it is from the first urn if the ball drawn is white?

(b) In a test an examinee either guesses or copies or knows the answer to a multiple choice question with 4 choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. Assume that the probability of answering a question correctly is unity for an examinee who knows the answer, $\frac{1}{4}$ for the examinee who guesses and $\frac{1}{8}$ for the examinee who copies. Compute the probability that an examinee knows the answer to a question, given that he has correctly answered it.

6 + 6 = 12

Group – E

8. (a) The probability mass function of a discrete random variable X is defined as
$P(X = 0) = 3k^2$, $P(X = 1) = 4k - 10k^2$, $P(X = 2) = 5k - 1$ and $(X = x) = 0$
if $x \neq 0, 1, 2$.

(i) Find the value of k.
(ii) Obtain the distribution function F(x) of X.
(iii) Find $P\{(0 < X < 2)/(X > 0)\}$.

(b) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson distribution with average number of demand per day 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.

7 + 5 = 12

9. (a) As a result of test on electric light bulbs, it was found that the lifetime of a particular make was distributed normally with an average life of 1000 burning hours and standard deviation of 200 hours. Out of 10,000 bulbs produced by the company how many bulbs are expected to fail (i) in the first 800 burning hours, (ii) between 800 and 1200 burning hours?

(b) The expenditure of 1000 families is given below :

Expenditure (Rs.)	40-59	60-79	80-99	100-119	120-139
Frequency	50	f_1	500	f_2	50

The median and mean for the distribution are both Rs. 87.50. Calculate the missing frequencies.

6 + 6 = 12