MATHEMATICAL METHODS IN CHEMICAL ENGINEERING
(CHEN 3204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The eigenvalues for the matrix $A=\begin{bmatrix}1 & -1\\ 2 & 4\end{bmatrix}$ are

(a) 3 and 2 (b) 3 and 1
(c) 2 and 1 (d) 4 and 3.

(ii) Legendre polynomial $P_4(x)$ will give us a _____ order polynomial.
(a) 5th (b) 3rd (c) 4th (d) none of these.

(iii) The radius of convergence for the series $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots$ is equal to

(a) 1 (b) 2 (c) ∞ (d) 0.5.

(iv) Determinant of a matrix A can be given by
(a) ∑(matrix element)x(corresponding cofactors)
(b) ∑(matrix element)x(corresponding minors)
(c) the ratio of adjoint of the matrix to the inverse of the matrix
(d) both (a) and (c).

(v) For a series function any value around a point 'a' the series converges is called
(a) convergence radius (b) midpoint of the convergence radius
(c) ordinary point (d) convergence intervals.

(vi) In a heat conduction problem the temperature varies with position and time according to T = T(x,y,t). The correct representation of the problem is
(a) $\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$ (b) $\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial y^2}$
(c) $\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$ (d) $\frac{\partial T}{\partial t}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$

(vii) Laplace transformation method is useful for solving PDEs because
(a) it reduces the number of variables
(b) it converts PDEs into ODEs
(c) it converts PDEs into algebraic equations
(d) it can be solved numerically.

(viii) Boundary layer separation for a flow in x direction will occur at a point if at that point
(a) $\partial P/\partial x > 0$ (b) $\partial P/\partial x < 0$ (c) $\partial P/\partial x = 0$ (d) $\partial P/\partial x = \infty$.

(ix) Navier-Stokes equation is an example of
(a) linear PDE (b) linear ODE
(c) non-linear PDE (d) non-linear ODE.

(x) δ/l for a boundary layer is proportional to
(a) $1/Re$ (b) Re (c) $1/\sqrt{Re}$ (d) $Re^{3/2}$

Group – B

2. (a) Evaluate A^5 using Cayley-Hamilton theorem, where $A=\begin{bmatrix}-1 & 3\\ -2 & 4\end{bmatrix}$.

(b) For a distillation column after doing component balance against the column under steady state condition, one derives simultaneous equations, which has the coefficient matrix $A=\begin{bmatrix}1 & -1 & 2\\ -1 & 3 & 0\\ 2 & 0 & 2\end{bmatrix}$. Comment on the obtaining solution of the system using rank analysis. Find out the eigenvector for the matrix using cofactors.

6 + (3 + 3) = 12

3. For a chemical system having components 1, 2, 3 and 4, the mass balance equations against a unit operation is given as below.
$2x_1-3x_2-2x_4+x_5=8$; $3x_2+2x_3+x_4=5$; $x_1-2x_2-4x_3+2x_4+2x_5=2$; $2x_1+x_2-3x_3-x_4+x_5=6$;
Where, 'x_n' for n=1,2,3,4 are the mole fractions. Comment on the degrees of freedom for the system. Find out the inverse of this matrix using Gauss-Jordan elimination method and the mole fractions if one of the mole fraction is set to '0'.

(2 + 7 + 3) = 12

Group – C

4. (a) Solve the differential equation $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\alpha(\alpha+1)y=0$, α is a constant. Considering α=3, the boundary conditions are y(0)=1 and y(0.002)=2. Neglect higher order terms with order of error $E(O^5)$.

(b) Find out the point of singularity for the equation given in Q.4(a)

10 + 2 = 12

5. Porous cylindrical shaped pellets (length=L and radius=R) are used as catalyst for the reaction, where A is converted to B. The rate constant of the reaction is 'k min^{-1}' in a packed bed. Develop a concentration profile of A within the pellet under steady state condition and when $\frac{L}{2R} > 3$. Steady state concentration is given as $C_{A,S}$.

12

Group – D

6. (a) Frame an initial value problem involving heat conduction with appropriate conditions.

(b) Explain the concept of orthogonal functions giving a suitable example.

7 + 5 = 12

7. (a) A partial differential equation is given: $\partial y/\partial t = \beta\, \partial^2 y/\partial z^2$. How can you obtain a solution for y as y = f(t)g(z) by the method of separation of variables?

(b) Solve the above-mentioned equation by Laplace transformation method, given the conditions:

(i) at t = 0, $y = y_0$

(ii) at z = 0, $y = y_1$ Given: $L^{-1}\left(\frac{1}{s}\, e^{-k\sqrt{s}}\right) = erfc\left(\frac{k}{2\sqrt{t}}\right)$.

7 + 5 = 12

Group – E

8. Derive the Prandtl boundary layer equations for flow past a cylindrical body of length l. Assume that the boundary layer thickness is much much smaller than l.

12

9. What is the significance of Damkohler number in connection to diffusion with chemical reaction? Explain how boundary layer concept can be applied to describe the situation when diffusion takes place along with chemical reaction for an isothermal laminar flow along a flat plate.

12