2015

MATHEMATICS - II
(MATH 1201)

Time Alloted : 3 Hours ***Full Marks : 70***

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following : **[10×1=10]**

i) The particular integral of $(D^2 + 2)y = x^2$ is

(a) $\frac{1}{2}(x^2 - 1)$ (b) $\frac{1}{2}(1 - x^2)$

(c) $\frac{x^2}{2}$ (d) $(1 + x^2)$

ii) The order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \left\{y + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}}$$ are

(a) 4, 2 (b) 1, 2

(c) 2, 2 (d) 2, 4

iii) Let $I = \int_0^1 f(x)dx$ where $f(x) = x^n \log x$, $n>0$ then

(a) I is proper
(b) I is improper
(c) f(x) has infinite discontinuity at x = 0
(d) both (b) and (c)

iv) $L^{-1}\left\{\frac{24}{(p+1)^5}\right\} =$

(a) $\frac{24t^3}{e^t}$
(b) $\frac{24t^4}{e^t}$
(c) $\frac{t^4}{e^{-t}}$
(d) $\frac{t^4}{e^t}$

v) A directed straight line makes angles 60°, 45° with the axes of x and y respectively. What angle does it make with z-axis?

(a) 30°
(b) 45°
(c) 60°
(d) 90°

vi) A tree having no cut vertex is a graph of

(a) three vertices
(b) two vertices
(c) two edges
(d) three edges

vii) Direction cosines of the line joining the points (3,2,5) and (–1,3,2) is

(a) $\frac{4}{\sqrt{26}}, -\frac{1}{\sqrt{26}}, \frac{3}{\sqrt{26}}$
(b) $-\frac{4}{\sqrt{26}}, \frac{1}{\sqrt{26}}, -\frac{3}{\sqrt{26}}$
(c) 4, –1, 3
(d) – 4, 1, – 3

viii) The number of pendant vertices in a binary tree having n vertices is

(a) $\frac{1}{2}(n + 1)$ (b) $\frac{1}{2}(n - 1)$

(c) $\frac{1}{2}n(n+1)$ (d) n

ix) BFS and DFS algorithms are used to find

(a) a spanning tree of a weighted graph

(b) a spanning tree of an unweighted graph

(c) shortest distance betweend any two vertices

(d) isomorphisms of graph

x) The length of the directed normal from the origin to the plane $2x - 3y + 6z = 7$ is

(a) –1 (b) 1

(c) 2 (d) 7

GROUP - B

2. (a) Solve : $\frac{dy}{dx} + y = y^3 (\cos x - \sin x)$

(b) Solve : $x^2y\frac{d^2y}{dx^2} + \left(x\frac{dy}{dx} - y\right)^2 = 0$ **6+6 = 12**

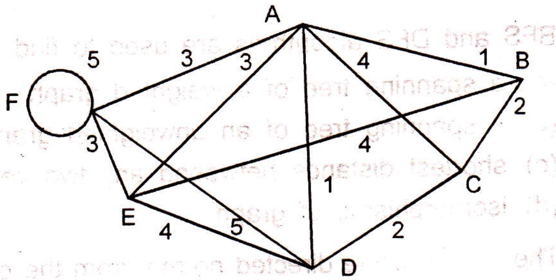
3. (a) Solve $\frac{d^2y}{dx^2} + 4y = \sin 2x$

(b) Apply the method of variation of parameters to solve the equation $\frac{d^2y}{dx^2} + 4y = 4 \sec^2 2x$ **4+8 = 12**

GROUP - C

4. (a) Is it possible to draw a graph with 4 vertices, 4 edges and with the degree sequence {1,2,3,4}? Justify your answer.

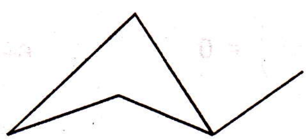
   (b) Find the minimal spanning tree and the corresponding weight, of the following weighted graph using Kruskal's Algorithm:

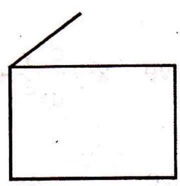


   (c) Let G be a bipartite graph of order 22 with partite sets U and W, where |U| = 12. Suppose that every vertex in U has degree 3; while every vertex of W has degree 2 or 4. How many vertices of G have degree 2?

3+6+3 = 12

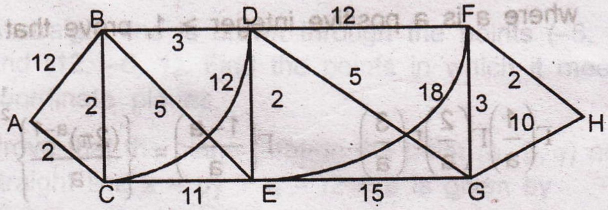
5. (a) What is meant by isomorphism between two graphs? Verify with proper reasoning whether the following graphs are isomorphic to each other.





   (b) If $\overline{G}$ be the complement of a simple graph G, find the value of d(v) in G + d(v) in $\overline{G}$.

(c) Find the shortest path between A and H and the distance along the shortest path using Dijkstra's Algorithm.



3+2+7 = 12

Group - D

6. (a) Check the convergence of the integral $\int_1^{\infty} \frac{dx}{\sqrt{x^2+x}}$ and justify your answer.

(b) Prove that $\frac{\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(2n)} = \frac{\sqrt{\pi}}{2^{2n-1}}$ (n>0)

(c) Evaluate L[f(t)], where $f(t) = \begin{cases} (t-1)^2, & t>1 \\ 0, & 0<t<1 \end{cases}$

3+6+3 = 12

7. (a) Using $\sin\frac{\pi}{a}\sin\frac{2\pi}{a}\sin\frac{3\pi}{a}\text{..........}\sin\frac{(a-1)\pi}{a}=\frac{a}{2^{a-1}}$, where a is a positive integer > 1, prove that

$$\Gamma\left(\frac{1}{a}\right)\Gamma\left(\frac{2}{a}\right)\Gamma\left(\frac{3}{a}\right)\text{..........}\Gamma\left(\frac{1-a}{a}\right)=\left\{\frac{(2\pi)^{a-1}}{a}\right\}^{\frac{1}{2}}$$

(b) Solve the following differential equation using Laplace Transformation

$$\frac{d^2x}{dt^2}+4x=\sin 3t \text{ where } x(0)=0,\ \dot{x}(0)=0$$

(c) Evaluate : $L^{-1}\{\tan^{-1}(p+2)\}$.

3+6+3 = 12

GROUP - E

8. (a) Show that the straight lines whose direction cosines are given by $a^2 l + b^2 m + c^2 n = 0$, $mn + nl + lm = 0$ are parallel if $a + b + c = 0$ (the direction cosines are denoted by l, m, n and a, b, c are any constant)

(b) Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1$, $x = 0$ and parallel to the line $\frac{x}{a}-\frac{z}{c}=1$, $y = 0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d is the shortest distance prove that $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{d^2}$

6+6 = 12

9. (a) Find the image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$.

(b) A straight line is drawn through the points (–6, 6, –5) and (12, –6, 1). Find the points in which it meets the coordinate planes.

(c) Prove that the plane through the point (α, β, γ) and the straight line $x = py + q = rz + s$ is given by

$$\begin{vmatrix} x & py+q & rz+s \\ \alpha & p\beta+q & r\gamma+s \\ 1 & 1 & 1 \end{vmatrix} = 0$$

3+5+4 = 12