M.TECH/CSE/1ST SEM/CSEN 5101/2018

ADVANCED DATA STRUCTURES
(CSEN 5101)

Time Allotted : 3 [illegible] **[illegible]ull Marks : 70**

Fig[illegible]

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) An infix arithmetic statement P is converted to its Polish postfix form Q. Each operand in P is a single lower case alphabetic character, and the operators in P all belong to the set of binary operators { =, +, -, *, /, ^ }. The operators have their usual meaning, = is the assignment operator and ^ is the exponentiation operator. In addition, P can contain one or more pairs of open and closed parentheses. Suppose P is 25 characters long (with no embedded blanks) and contains three pairs of open and closed parentheses. In this case, how many operator symbols occur in Q?
(a) 12 (b) 11 (c) 10 (d) 9.

(ii) A binary tree T having 17 nodes is stored in a two-dimensional table BINTREE[17][3]. The first row of BINTREE corresponds to the root node of T, [illegible]r nodes of T. The firs[illegible]er key value, the second column contains the row number of the left child, and the third column contains the row number of the right child. Null links are represented by -1. Of the total 51 entries in BINTREE, how many are not equal to -1?
(a) 27 (b) 30 (c) 33 (d) 36.

(iii) Let S represent an instance of the Stack ADT. Let S.push(x) push the value x on to the top of the stack, S.pop() remove the topmost element from the stack and return the value. Consider the following sequence of operations performed on S which initially contains 10 elements with 55 as the topmost element (assuming that S is of sufficient capacity).
S.push(7); S.push(20); S.push(35); S.pop(); S.push(14); S.pop(); S.pop();
What will be the element at the top of the stack after the above sequence of operations?
(a) 7 (b) 14 (c) 35 (d) 55.

(iv) Suppose you have the following hash table, implemented using linear probing. The hash function is h(x) = x mod m, where m is the size of the hash table.

0	1	2	3	4	5	6	7	8
9	18		12	3	14	4	21	

In which of the following order could the elements have been added to the hash table? Assume that the hash table has never been resized, and no elements have been deleted yet.
A. 9, 14, 4, 18, 12, 3, 21 B. 12, 3, 14, 18, 4, 9, 21
C. 12, 14, 3, 9, 4, 18, 21 D. 9, 12, 14, 3, 4, 21, 18
E. 12, 9, 18, 3, 14, 21, 4.
(a) only A (b) D & E (c) C & D (d) only B.

(v) The following array is sorted using insertion sort:
10 20 30 50 40 60 70 90 80 100
How many pairwise comparisons are made?
(a) 12 (b) 11 (c) 10 (d) 9.

(vi) What is the maximum possible black height of a Red Black (RB) tree that has 12 nodes?
(a) 2 (b) 3 (c) 4 (d) 5.

(vii) Suppose there is an AVL tree of height 4.What is the minimum possible number of nodes that can be present in the tree?
(a) 12 (b) 13 (c) 11 (d) 10.

(viii) Consider the max heap represented by the following array:

Array Index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now suppose that a new value 35 is inserted into this heap. After insertion, the new heap is
(a) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35 (b) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
(c) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15 (d) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30.

(ix) A binary search tree T is first traversed in preorder and then in postorder. It is found that the sequence in which the nodes are visited is identical in the two cases. Then it must be the case that
(a) no node in T has a left child
(b) no node in T has a right child
(c) every internal node in T has two children
(d) T just has a root node and no other node.

(x) We want to search for a given string P of n characters in another string T of m characters, where n is much smaller than m. If a naive (brute-force) pattern matching method is used, the search will take a worst-case time roughly proportional to
(a) m (b) n (c) m.n (d) m+n.

Group – B

2. (a) An array A containing 18 positive integers is to be converted into a max-heap in 17 or fewer pairwise comparisons among the elements of A. Is this always possible? Explain giving reasons. Use the 18-element array given below for illustration.
Array A: 8 17 2 35 12 42 11 14 3 65 34 7 19 5 6 16 54 60

(b) Two claims are made below which might be TRUE or FALSE. For each claim, state which is the case giving reasons.

CLAIM 1: It is always possible to convert an array A of n positive integers into a max-heap in less than $((\text{int})\lg n)^2$ pairwise comparisons among the elements of A (here int refers to the integer part).

CLAIM 2: An array A of 19 positive integers is already a max-heap. The element at the root (i.e., at A[0]) is deleted and a new element of value 0 is inserted at that position. The claim is that this new array can always be converted into a max-heap in no more than 5 pairwise comparisons.

6 + (3 + 3) = 12

3. (a) Insert the following 13 keys in the given sequence into a hash table TAB of size 17 (i.e., the table starts at TAB[0] and goes up to TAB[16]) using the open addressing method with linear probing:
18, 15, 92, 44, 58, 26, 13, 24, 37, 74, 66, 82, 38
Determine the average number of comparisons for a successful search.

(b) What is meant by primary clustering? How does it differ from secondary clustering? Explain with reference to the hash table of part (a) above.

(4 + 2) + (3 + 3) = 12

Group – C

4. (a) Answer the parts below giving reasons in each case.

How many distinct binary search trees, containing the three keys 10, 17 and 28, will all yield the traversal sequence 10 17 28 when traversed in (i) pre-order; (ii) in-order; (iii) post-order?

(b) (i) What is meant by the balance factor of a node in a binary tree?

When is a binary tree said to be balanced? Consider all binary trees that have five nodes. How many of these are balanced?

(ii) A binary search tree contains the seven keys 10, 20, 30, 40, 50, 60, 70. If T has height 3, which keys can possibly occur at the root of T? If T has height 4, which keys can possibly occur at the root of T?

(2 + 2 + 2) + (3 + 3) = 12

5. (a) Consider the following set S of 14 keys:
S = { 2, 5, 6, 8, 10, 15, 18, 22, 37, 38, 44, 46, 47, 49 }
Using the above set of keys, construct: (i) an AVL tree of height = 4; (ii) a Red-Black (RB) tree of black height = 2, clearly marking the red and black nodes.

(b) (i) Give an example of a Red-Black (RB) tree T of black height = 3, which, when the node colours are erased, is not an AVL tree.

(ii) Explain how any given AVL tree can be converted into a Red Black (RB) tree by appropriately colouring the nodes.

(3 + 3) + (3 + 3) = 12

Group – D

6. (a) Define a B-Tree. When does a B-Tree become a 2-3 Tree?

(b) Show the B-tree that results when inserting the keys R,Y,F,X,A,M,C,D,E,T,H,V,L,W,G (in the given sequence using lexicographic ordering). Assume a minimum branching factor of t = 3. Show the B-tree just before and after a node split as keys get inserted.

(3 + 2) + 7 = 12

7. (a) Given a sorted list of n keys, how do we construct a skip list that ensures the average time to search a key in the list is proportional to lg n? Use the following sorted list of 21 keys to illustrate the construction method:
21 25 33 39 46 51 56 62 68 73 81 85 89 92 97 101 110 115 118 125 134

(b) We now want to insert the new key 49 into the skip list of part (a). Suggest a randomized method of insertion that helps to maintain the property that the average search time is proportional to lg n.

How can we take care of the deletion of a key from the skip list?

6 + (4 + 2) = 12

Group – E

8. (a) What role does the prefix function play in the KMP pattern matching algorithm?

   (b) Suppose that the procedure 'Compute-Prefix-Function (P)' is available to you. Write the pseudo-code for a KMP matcher (T, P) for text T and pattern P using the above procedure.

   (c) Comment on the time complexity of the KMP matcher that you have written in part (b) above with proper justification.

3 + 6 + 3 = 12

9. (a) This question relates to computational geometry. Let Q = { p_1, p_2, p_3, p_4 } be a set of 4 points in the X-Y plane.
   We want to determine whether the line segment joining p_1 and p_2 intersects the line segment joining p_3 and p_4. Describe briefly a method that can be used to solve the problem. For illustration, use the points $p_1 = (10,5)$, $p_2 = (5,10)$, $p_3 = (6,8)$, $p_4 = (15,3)$.

   (b) This time we want to determine the convex hull of the four points in Q. Describe briefly a method for solving the problem, using the points in Q for illustration.

6 + 6 = 12