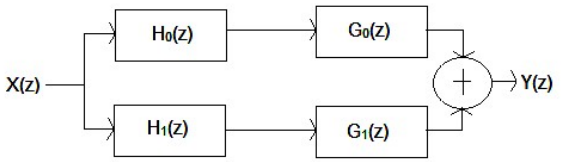
filters $G_0(z)$ and $G_1(z)$ so that the structure of figure below is a perfect reconstruction filter bank.



(b) Let the analysis filters $H_0(z)$ and $H_1(z)$ of the above structure be the power complementary FIR filters of order N each. Show that this structure becomes a perfect reconstruction filter bank if the synthesis filters $G_0(z)$ and $G_1(z)$ are chosen as

$$G_0(z) = z - NH_0(z^{-1}) \text{ and } G_1(z) = z - NH_1(z^{-1}).$$

6 + 6 = 12

7 (a) What do you mean by adaptive filters? Explain with basic adaptive filter structure.

(b) Derive weight update expression of an adaptive filter using LMS algorithm.

(c) Explain how adaptive filters are used in system identifications.

3 + 5 + 4 = 12

Group - E

8. (a) What is the need for spectral estimation? How can the energy density spectrum be determined?

(b) A discrete-time signal x(n) is given as under:

$$x(n) = \cos(2\pi f_1 n) + \cos(2\pi f_2 n),\ n = 0,1,2,....,7.$$

Determine the power spectrum for the data sequence length $L = 8,16$ for different values of f_1 and f_2, where $f_2 = f_1 + \Delta f$ and Δf is small deviation from f_1 or simply frequency separation.

(2 + 2) + 8 = 12

9. (a) What are AR and ARMA processes?

(b) A 1st order auto regressive (AR) process is given by $x(n) = Ax(n-1) + \omega(n)$, where A is a constant and $\omega(n)$ is a white noise process of zero mean and variance σ^2. Find the mean and autocorrelation function of the process $x\{x(n)\}$.

(2 + 2) + 8 = 12

ADVANCED DIGITAL SIGNALS AND SYSTEMS (AEIE 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Condition for aliasing problem is:
(a) $f_s < f_m$ (b) $f_s < 2f_m$
(c) $f_s = f_m$ (d) all of these.

(ii) Which of the following system is causal?
(a) $y(n) = x(-n) - x(2n-1)$ (b) $y(n) = x^2(n) - x(n+1)$
(c) $h(n) = n\left(\frac{1}{2}\right)^n u(n)$ (d) $h(n) = n\left(\frac{1}{2}\right)^n u(n+1)$.

(iii) Which of the following substitution is done in Bilinear transformations?
(a) $s = \frac{2}{T}\left[\frac{1+z^{-1}}{1-z^{-1}}\right]$ (b) $s = \frac{2}{T}\left[\frac{z^{-1}+1}{z^{-1}-1}\right]$
(c) $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ (d) None of the above.

(iv) A continuous-time signal $x_a(t)$ is known to be uniquely recoverable from its samples $x_a(nT_s)$ when $T_s = 1$ ms. What is the highest frequency in $x_a(f)$?
(a) 5000 Hz (b) 1000 Hz
(c) 500 Hz (d) none of these.

(v) If $DFT\{x(n)\} = X(k)$, then $DFT\{x(n+m)_N\}$

(a) $X(k)e^{-j\frac{2\pi km}{N}}$ (b) $X(k)e^{-j\frac{2\pi k}{mN}}$ (c) $X(k)e^{j\frac{2\pi km}{N}}$ (d) $X(k)e^{j\frac{2\pi k}{mN}}$.

(vi) For a system $y(n) = x(n-3)$, the impulse response of the system and the inverse system will be,

(a) $h(n) = \delta(n+3)$ and $x(n) = y(n-3)$ respectively

(b) $h(n) = \delta(3n)$ and $x(n) = y\left(\frac{n}{3}\right)$ respectively

(c) $h(n) = \delta(n-3)$ and $x(n) = y(n+3)$ respectively

(d) $h(n) = \delta(n+3)$ and $x(n) = y(3n)$ respectively.

(vii) Which of the following is the difference equation of the FIR filter of length M, input $x(n)$ and output $y(n)$?

(a) $y(n) = \sum_{k=0}^{M+1} b_k x(n+k)$ (b) $y(n) = \sum_{k=0}^{M+1} b_k x(n-k)$

(c) $y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$ (d) None of the above.

(viii) Which operation is implemented by keeping every M-th sample of $x(n)$ and removing $M-1$ in between samples to generate $y(n)$?

(a) Up-sampling (b) Down-sampling

(c) Both (a) and (b) (d) None of the above.

(ix) If $X_1(k)$ and $X_2(k)$ are the N-point DFTs of $x_1(n)$ and $x_2(n)$ respectively, then what is the M-point DFT of $x(n) = ax_1(n) + bx_2(n)$?

(a) $X_1(ak) + X_2(bk)$ (b) $aX_1(k) + bX_2(k)$

(c) $e^{ak}X_1(k) + e^{bk}X_2(k)$ (d) None of the mentioned.

(x) $E[X - E(X)]$ is equal to:

(a) $E(X)$ (b) $\mathrm{Var}(X)$ (c) 0 (d) $E(X) - X$.

Group - B

2. (a) Determine whether the following system is time-invariant or time variant:

$$y(n) = x(-n)$$

(b) Represent the sequence $x(n) = \{2, \underset{\uparrow}{4}, -1, -2\}$ as sum of shifted unit impulses.

(c) Determine whether the following signal is energy or power signal:

$$x(n) = \left(\frac{3}{8}\right)^n u(n).$$

5 + 2 + 5 = 12

3. (a) Test the stability of the system whose impulse response

$$h(n) = \left(\frac{1}{2}\right)^n u(n).$$

(b) Find the circular convolution of two sequences $x_1(n) = \{1, 2, 3\}$ and $x_2(n) = \{1, -1, -2\}$ using DFT-IDFT method.

4 + 8 = 12

Group - C

4. (a) Design a lowpass Butterworth IIR digital filter using bilinear transformation technique with $T = 1$, from its analog filter that satisfies the following specifications:

Passband cutoff: $\Omega_p = 0.2\pi$; Passband ripple: $R_p = 7dB$

Stopband cutoff: $\Omega_s = 0.3\pi$; Stopband ripple: $A_s = 16\,dB$.

(b) Transform the analog filter $H_a(s) = \frac{s+1}{s^2 + 5s + 6}$ into a digital filter $H(z)$ using the impulse invariance technique in which $T = 0.1$.

8 + 4 = 12

5. (a) State the main disadvantage of direct form realization of digital filters.

(b) Obtain the direct form and cascade form realization for the transfer function of an FIR system given by

$$H(z) = \left(1 - \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1} - \frac{1}{2}z^{-2}\right).$$

2 + 10 = 12

Group - D

6. (a) Consider the analysis- synthesis filter bank shown in figure below. Develop the input output relation of this structure in the z-domain. Let $H_0(z) = \frac{(1+z^{-1})}{2}$ and $H_1(z) = \frac{(1-z^{-1})}{2}$. Determine the synthesis