(vii) The travelling salesperson problem can be solved in
(a) polynomial time using dynamic programming
(b) polynomial time using branch and bound
(c) polynomial time using backtracking
(d) exponential time using dynamic programming.

(viii) Let X be a problem that belongs to the class NP. Then which one of the following is true?
(a) There is no polynomial time algorithm for X
(b) If X can be solved deterministically in polynomial time, then P=NP
(c) If X is NP-hard, then it is NP-complete
(d) X may be undecidable.

(ix) What is the minimum number of colours that is sufficient to vertex-colour any planar graph?
(a) 2 (b) 3 (c) 4 (d) 1.

(x) Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?
(a) R is NP-complete (b) R is NP-hard
(c) Q is NP-complete (d) Q is NP-hard.

Group – B

2. (a) Solve the following recurrence using the master theorem:
$T(n) = 4T(n/2) + n^2$.

(b) Establish the theoretical lower bound of time complexity for any sorting algorithm, where the sorting is performed by pair-wise comparison of elements.

(c) Illustrate the operation of counting sort on the array A = {6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2}.

2 + 5 + 5 = 12

3. (a) With proper justification arrange the following functions in increasing order of growth

2^n $n^{3/2}$ nlogn $n^{\log n}$

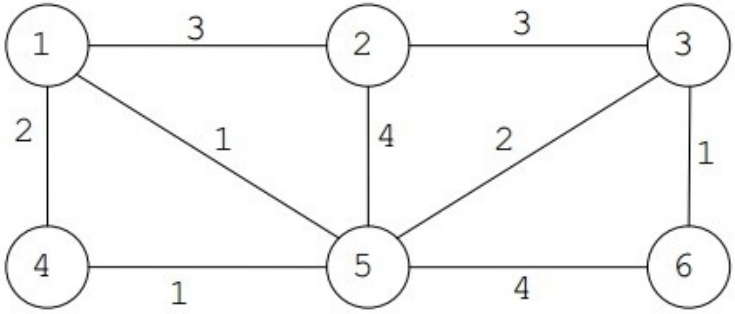
(b) Write a divide–and–conquer merge sort algorithm and derive the time complexity of this algorithm.

4 + 8 = 12

Group – C

4. (a) What is a fractional knapsack problem?

(b) Find a minimum spanning tree of the graph given bellow using Prim's algorithm.



(c) A set of seven jobs is given, with associated profit {3, 5, 20, 18, 1, 6, 30} and deadline {1, 3, 4, 3, 2, 1, 2} respectively, where the profit is earned if and only if a job is completed by its deadline. Only one machine is available for processing jobs and each job requires one unit of time on the machine to be completed. Find a sequence of jobs to be processed so that the profit is maximized.

2 + 5 + 5 = 12

5. (a) What is the time complexity of Floyd-Warshall algorithm?

(b) Using the given edge cost matrix for the vertex set {1, 2, 3, 4}, find a dynamic programming solution of the travelling salesperson problem, where the origin is vertex 1.

0	5	6	8
10	0	13	8
15	9	0	9
20	10	12	0

(c) Will Bellman Ford algorithm be able to find shortest paths from a given source vertex to all the other vertices in a directed weighted graph if the graph contains a *negative* edge weight, but no negative cycle? How is a negative cycle detected by Bellman Ford algorithm?

2 + 6 + (3 + 1) = 12

Group – D

6. (a) What is n-queens problem? With the help of a state space tree explain how backtracking is used to solve 4-queens problem.

(b) Write an algorithm to find the number of (connected) components of a graph.

(1 + 5) + 6 = 12

7. (a) Explain naïve string matching algorithm with an example. What is the time complexity of the algorithm?

(b) What is an articulation point? Given an undirected connected graph G, explain how to find all articulation points in it.

6 + (1 + 5) = 12

Group – E

8. (a) Explain live node, e-node and dead node of a state-space graph in terms of branch and bound with suitable examples.

(b) Write the least-cost search method to find the solution of the 15 puzzle problem. Will this method guarantee solution for all instances of the problem?

6 + 6 = 12

9. (a) Explain Hamiltonian cycle of a weighted graph.

(b) What is a clique? Prove that the 'Clique Decision problem' is NP-complete.

3 + (2 + 7) = 12

DESIGN AND ANALYSIS OF ALGORITHMS
(MCAP 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Let W(n) and A(n) denote respectively, the worst case and average case running time of an algorithm executed on an input of size n. Which of the following is always true?
(a) $A(n) = \Omega(W(n))$ (b) $A(n) = O(W(n))$
(c) $A(n) = \Theta(W(n))$ (d) $A(n) = o(W(n))$.

(ii) Which of the following is not $O(n^2)$?
(a) $(15^{10})\, n + 12099$ (b) $n^{1.98}$ (c) $n^3 / n^{½}$ (d) $2^{20}\, n$.

(iii) In Knuth-Morris-Pratt pattern matching, the failure function for the pattern *aaab* is
(a) 1 2 3 0 (b) 1 1 1 0 (c) 0 1 2 0 (d) 0 1 2 1.

(iv) A union-find data structure is commonly applied while
(a) implementing a depth first search traversal of a graph
(b) implementing a breadth first search traversal of a graph
(c) computing the minimum spanning tree of a graph using the Kruskal algorithm
(d) computing the all pair shortest paths in a graph.

(v) A sorting technique is called stable if
(a) it takes O(n log n) time
(b) it takes O(n log n) space
(c) it maintains the relative order of occurrence of non-distinct elements
(d) it does not maintain the relative order of occurrence of non-distinct elements.

(vi) An all pair shortest paths problem is efficiently solved using
(a) depth first search (DFS) (b) Dijkstra's algorithm
(c) Bellman-Ford algorithm (d) Floyd-Warshall algorithm.