(b) Reduce the following game to a 2×2 game graphically and hence solve it.

	PLAYER B				
PLAYER A	2	−1	5	−2	6
	−2	4	−3	1	0

6 + 6 = 12

Group - D

6. (a) Consider the function $f(x,y) = 2xy - x^4 - x^2 - y^2$. Find the local optima point, if any, of the function. Determine also whether the local optima are global or not.

(b) Solve the following non-linear programming problem using Lagrange multiplier method:
Minimize $f(x_1, x_2, x_3) = 2x_1^2 + 2x_2^2 + 2x_3^2 - 24x_1 - 8x_2 - 12x_3 + 200$
subject to the constraints

$$x_1 + x_2 + x_3 = 11$$
$$x_1, x_2, x_3 \geq 0$$

4 + 8 =12

7. Maximize $f(x_1, x_2, x_3) = 4x_1 + 6x_2 - x_1^2 - x_2^2 - x_3^2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$2x_1 + 3x_2 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$

by applying Kuhn-Tucker conditions.

12

Group - E

8. Solve the following unconstrained optimization problem using Fibonacci Search method:

$$\min_x \left(\frac{1}{4}\right)x^4 - \left(\frac{5}{3}\right)x^3 - 6x^2 + 19x - 7$$

The initial interval of uncertainty is $[-4, 0]$ and the required length of interval of uncertainty is 0.2.

12

9. Write the Dichotomous Search Algorithm for unimodal functions of one variable, and using the algorithm, find the minimum of *x(x-1.5)* in the interval (0, 1) to within 10% of the exact value.

12

OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES (MATH 4181)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The set S given by S={(x,y): $x^2+y^2 \leq 25$} is
(a) convex (b) open
(c) non-convex (d) unbounded.

(ii) If the dual has unbounded solution, primal has
(a) unbounded solution (b) feasible solution
(c) no feasible solution (d) unique solution.

(iii) The solution to a transportation problem with m-rows and n-columns is feasible if number of positive allocations are
(a) $m - n + 1$ (b) $m + n$
(c) $m + n - 1$ (d) all of the above.

(iv) An LPP with '≥' type constraints can be solved by using
(a) Simplex Method (b) Graphical Method
(c) Big-M Method (d) North West Corner Rule.

(v) Assignment problem is solved by
(a) North West Corner Rule (b) Simplex Method
(c) Vogel's Approximation Method (d) Hungarian Method.

(vi) The function $f(x,y) = x^2y + y^2 + 2y$ has
(a) a local maximum point (b) a global maximum point
(c) a local minimum point (d) a saddle point.

(vii) The quadratic form $Q(x,y,z) = x^2 + 2xz + 2y^2 - z^2$ is
(a) positive definite (b) negative definite
(c) positive semidefinite (d) indefinite

(viii) The value of the following game problem

PLAYER B

PLAYER A

−2	2
−1	4
2	3

is

(a) 4 (b) 3 (c) 2 (d) -1.

(ix) If (1, 2) is a stationary point of the function f(x, y),which one of the following conditions assures that (1, 2) is a global maximum point of the function?
(a) Hessian matrix of the function f(x, y) is negative definite at (1,2)
(b) Hessian matrix of the function f(x, y) is negative definite for all (x, y)
(c) Hessian matrix of the function f(x, y) is positive definite for all (x, y)
(d) Hessian matrix of the function f(x, y) is positive definite at (1, 2).

(x) For an unimodal function
(a) the global optima does not exist
(b) the local and global optima are not same
(c) an extreme point is a global optima
(d) multiple extreme points exist.

Group – B

2. (a) Use Simplex method to solve the following LPP :
Maximize $z = x_1 + x_2 + 4x_3 + 2x_4$
Subject to $x_1 + 3x_3 + x_4 \leq 4$
$x_3 + 2x_4 \leq 3$
$x_1 + 4x_2 + x_3 \leq 3$
$x_1, x_2, x_3, x_4 \geq 0$

(b) Find the dual of the following LPP :
Maximize $z = 2x_1 + 3x_2 - 4x_3$
Subject to $3x_1 + x_2 + x_3 \leq 2$
$-4x_1 + 3x_3 \geq 4$
$x_1 - 5x_2 + x_3 = 5$
$x_1, x_2 \geq 0; x_3$ is unrestricted in sign.

9 + 3 = 12

3. (a) Use the 'Big-M' method to solve the following LPP:
Maximize $z = 2x_1 + 3x_2$
Subject to $3x_1 + 5x_2 \geq 30$
$5x_1 + 3x_2 \geq 60$
$x_1, x_2 \geq 0$

(b) Write the dual of the following LPP:
Minimize $z = 2x_1 + 2x_2$
Subject to $2x_1 + 4x_2 \geq 1$
$x_1 + 2x_2 \geq 1$
$2x_1 + x_2 \geq 1$
$x_1, x_2 \geq 0$

8 + 4 =12

Group - C

4. (a) Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method:

	W_1	W_2	W_3	W_4	Capacity
F_1	10	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

(b) A salesman has to visit five cities A, B, C, D and E. The distance (in hundred miles) between the five cities are as follows:

	A	B	C	D	E
A	-	7	6	8	4
B	7	-	8	5	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

If the salesman starts from city A and has to come back to city A, which route should he select so that the total distance travelled is minimum?

6 + 6 = 12

5. (a) Use dominance to reduce the following game problem to 2 × 2 game and hence find the optimal strategies and the value of the following game:

PLAYER B

PLAYER A

3	2	4	0
3	4	2	4
4	2	4	0
0	4	0	8