NUMERICAL AND STATISTICAL METHODS
(MATH 2002)

Time Allotted : 3 hrs   **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

   (i) The relative error in approximating $\frac{2}{3}$ to 0.6667 is
   (a) 5×10^{-5} (b) 5×10^{-6} (c) 5×10^{-3} (d) 3.3×10^{-5}.

   (ii) If $f(x) = ax^2$ (a is constant) and $h = 1$, then which one is wrong
   (a) $\Delta f(x) = a(2x + 1)$ (b) $\Delta^2 f(x) = 2a$
   (c) $\Delta^3 f(x) = 2$ (d) $\Delta^4 f(x) = 0$

   (iii) The iterative formule to find the the mth root of a positive real number R by Newton-Raphson method is
   (a) $\frac{(m-1)x_n^m + R}{mx_n^{m-1}}$ (b) $\frac{mx_n^m + R}{mx_n^{m-1}}$ (c) $\frac{x_n^m + R}{mx_n^{m-1}}$ (d) $\frac{(m+1)x_n^m + R}{mx_n^{m-1}}$.

   (iv) If X and Y are independent variates then correlation coefficient of X and Y is
   (a) 1   (b) 0   (c) −1   (d) 2 .

   (v) A purse contains 4 copper coins, 3 silver coins and another purse contains 6 copper coins, 2 silver coins. A purse is chosen at random and a coin is taken out of it. The probability that it is a copper coin is
   (a) $\frac{4}{7}$ (b) $\frac{3}{4}$ (c) $\frac{3}{7}$ (d) $\frac{37}{56}$.

   (vi) If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$, then $P(A^C \cap B^C)$ is
   (a) $\frac{5}{12}$ (b) $\frac{1}{12}$ (c) $\frac{3}{4}$ (d) $\frac{7}{12}$.

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7. (a) The probability of detecting tuberculosis in X-ray examination of a person suffering from the disease is $(1-b)$. The probability of diagnosing a healthy person as tubercular is a. If the ratio of tubercular patients to the whole population is c, find the probability that a person is healthy if after examination he is diagnosed as tubercular.

   (b) Two integers x, y are chosen at random with replacement from nine natural numbers {1,2,3,4,5,6,7,8,9}. Find the probability that $\left|x^2 - y^2\right|$ is divisible by 2.

   (c) X can hit a target 4 times out of 5, Y can hit 3 times out of 4 and Z can hit 2 times out of 3. Each person is allowed one attempt. What is the probability that the target is hit? What is the probability that the target is hit exactly two times?

   6 + 2 + 4 = 12

Group - E

8. (a) If the probability density function of a random variable X is defined by, $f(x) = kx^2 e^{-x}$, $x \geq 0$
   $= 0$, otherwise
   Then find (i) k , (ii) $E(X)$, and (iii) $Var(2 - 3X)$.

   (b) A point P is chosen at random on a straight line segment AB of length 2a. Find the probability that area of the rectangle with sides AP and PB will exceed $\frac{a^2}{2}$.

   6 + 6 = 12

9. (a) A chip is being considered for use in a certain system. The lifetime of the chip is modeled by a Normal random variable with mean 20,000 hours and standard deviation 5000 hours. Find the probability that the lifetime of a chip is between 18,000 hours and 22,000 hours.

   (b) The median and mode of the following wage distribution of 230 employees are known to be Rs. 3350 and Rs. 3400 respectively. Find f_3, f_4, f_5.

Wages(Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000	6000-7000
No. of employees	4	16	f_3	f_4	f_5	6	4

   (c) If $4x + y = 52$ and $x + y = 32$ be two regression lines, then calculate mean of X and mean of Y.

   4 + 6 + 2 = 12

(vii)   The random variable X has the distribution $F(0) = \frac{1}{6}, F(1) = \frac{1}{2}, F(3) = \frac{3}{4}$,

then $P(0 < X \leq 3)$ is

(a) $\frac{5}{12}$ (b) $\frac{1}{12}$ (c) $\frac{3}{4}$ (d) $\frac{7}{12}$.

(viii) The percentage error in approximating $\frac{4}{3}$ to 1.3333 is

(a) 0.0025   (b) 0.0035   (c) 0.25   (d) 0.025.

(ix)   The accuracy of Simpson's quadrature for a step size h is

(a) $o(h^2)$ (b) $o(h^4)$ (c) $o(h^3)$ (d) $o(h^5)$.

(x)   Which of the following is a predictor-corrector method ?
(a) Euler's method
(b) Modified Euler's method
(c) Runge-Kutta method of 4th order
(d) None of the options given.

Group – B

2. (a) Find a real root of the equation $x^3 + 2x - 2 = 0$ by Regula-Falsi method, correct upto two decimal places.

(b) Use LU factorization method to solve the following system of linear equations
$$3x + 2y - 4z = 12$$
$$-x + 5y + 2z = 1$$
$$2x - 3y + 4z = -3$$

5 + 7 = 12

3. (a) Solve the following system of equations by Gauss-Elimination method:
$$x + 2y + z = 0$$
$$2x + 2y + 3z = 3$$
$$-x - 3y = 2$$

(b) Use Newton-Raphson method to derive an iterative scheme to evaluate $\sqrt[k]{n}$. Hence, use the scheme to find $\sqrt[4]{12}$, correct up to two significant figures.

6 + 6 = 12

Group – C

4. (a) Find the area bounded by the curve $f(x)$ and x-axis between $x = 7.47$ and $x = 7.52$ from the following table:

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

(b) Use the Lagrange's method to find a polynomial which approximates the following data. Hence, use it to find $f(0)$.

x	-1	-2	2	4
$f(x)$	-1	-9	11	69

6 + 6 = 12

5. (a) Find the solution of the differential equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1$ for $x = 0.08$ taking $h = 0.02$ using Euler's method correct up to 4 places of decimal.

(b) Evaluate $\int_0^{0.6} e^x dx$ by using Simson's $\frac{1}{3}$rd rule taking n = 6.

(c) Find the value of $y(0.6)$ using Runge-Kutta method of 4th order, given that $\frac{dy}{dx} = \sqrt{x+y}$, $y(0.4) = 0.41$ taking h = 0.2.

4 + 4 + 4 = 12

Group - D

6. (a) A computer manufacturer uses chips from three sources. Chips from sources A, B and C are defective with probabilities 0.005, 0.001 and 0.010 respectively. If a randomly selected chip is found to be defective, find the probability that the manufacturer was A. Assume that the proportions of chips from A, B and C are 0.5, 0.1 and 0.4 respectively.

(b) A and B are two independent witnesses. The probability that A will speak the truth is x and probability that B will speak the truth is y. Suppose A and B agree in a certain statement. Show that the probability of the statement being true is $\frac{xy}{1-x-y+2xy}$.

6 + 6 = 12