MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If $f(z) = C_0 + C_1 z$, then $\oint \frac{1+f(z)}{z} dz$ over unit circle is given by

(a) $2\pi C_1$ (b) $2\pi(1 + C_0)$ (c) $2\pi i C_1$ (d) $2\pi i(1 + C_0)$.

(ii) For the function $f(z) = \frac{\sin z}{z}$, $z = 0$ is

(a) an essential singularity (b) a simple pole
(c) a pole of order 2 (d) a removable singularity.

(iii) If a function $f(x) = x^2, -\pi \le x \le \pi$ is represented by a Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then the value of b_n is

(a) $\frac{2\pi^2}{3}$ (b) $\frac{4(-1)^n}{3}$ (c) 0 (d) 1.

(iv) The value of $\int_{-1}^{1} \{P_n(x)\}^2 dx$ is

(a) 1 (b) $\frac{2}{2n+1}$ (c) $2n^2$ (d) 0.

(v) $J_{\frac{1}{2}}^2(x) + J_{-\frac{1}{2}}^2(x) =$

(a) 10 (b) 0 (c) $\frac{2}{\pi x}$ (d) $\frac{2}{\pi x}\sin 2x$.

(vi) If $\mathcal{F}\{f(t)\} = F(s)$ and $a \neq 0$, then $\mathcal{F}\{f(at)\} =$

(a) $\frac{1}{a}F(\frac{s}{a})$ (b) $\frac{1}{a}F(\frac{a}{s})$ (c) $\frac{1}{|a|}F(\frac{a}{s})$ (d) $\frac{1}{|a|}F(\frac{s}{a})$.

(vii) The value of $P_1(x)$ is

(a) 1 (b) x (c) 0 (d) $\frac{4}{7}$.

(viii) $\frac{d}{dx}J_0(x) =$

(a) $-J_1(x)$ (b) $J_1(x)$ (c) 0 (d) $\frac{2}{x} - J_1(x)$.

(ix) If $F(s)$ denotes the Fourier transform of $f(t)$, then the Fourier transform of $tf(t)$ is

(a) $\frac{d}{ds}\{F(s)\}$ (b) $i\frac{d}{ds}\{F(s)\}$

(c) $-i\frac{d}{ds}\{F(s)\}$ (d) None of (a), (b), (c).

(x) Particular integral of the PDE $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2x + 3y$ is

(a) $\frac{1}{150}(2x+3y)^3$ (b) $\frac{1}{75}(2x+3y)^3$

(c) $\frac{1}{150}(2x+3y)^2$ (d) $\frac{1}{75}(2x+3y)^2$

Group – B

2. (a) Find the analytic function $f(z) = u + iv$ in terms of z, given $2u + v = e^x(\cos y - \sin y)$.

(b) (i) Show that an analytic function with constant modulus is constant.
(ii) Express $f(z) = \cos(z)$ in $u(x,y) + iv(x,y)$ form where $z = x + iy$.

6 +(4 + 2) = 12

3. (a) Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent series in the region $|z| < 1$.

(b) Find the pole of the function $f(z) = \frac{e^{iz}}{z^2+1}$ and the residues at each pole.

6 + 6 = 12

Group – C

4. (a) Find the half-range sine series of $f(x) = x\cos x$ in $0 < x < \pi$

(b) Obtain Fourier series for the function
$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$
Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$.

6 + 6 = 12

B.TECH/AEIE/CE/ ECE/ EE /3RD SEM/ MATH 2001/2018

5.(a) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$. Hence deduce Fourier sine transform of $g(x) = \frac{x}{1+x^2}$.

(b) Find the inverse Fourier transform of $\frac{1}{s^2+6s+25}$.

7 + 5 = 12

Group – D

6. (a) Obtain the power series solution of $xy'' + 2y' - xy = 0$ about $x = 0$.

(b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$

8 + 4 =12

7. (a) Prove that
$$\int_{-1}^{1} P_m(x)P_n(x)\,dx = 0, \quad \text{for } m \ne n.$$

(b) Prove that $\frac{d}{dx}\{x^{-n}J_n(x)\} = -x^{-n}J_{n+1}(x)$.

6 + 6 = 12

Group – E

8. (a) Solve : $z(x+y)p + z(x-y)q = x^2 + y^2$

(b) Solve the given partial differential equation using method of separation of variables : $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ given that $u(0,y) = 8e^{-3y}$.

6 + 6 = 12

9. (a) Find the complete integral of $pxy + pq + qy = yz$ by using Charpit's method.

(b) Find the general solution of the following partial differential equation $(D^2 + DD' - 6D'^2)z = y\cos x$

6 +6 =12