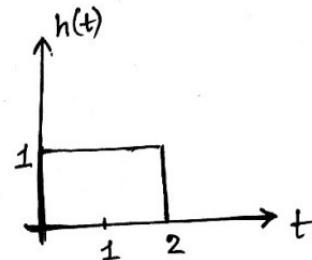
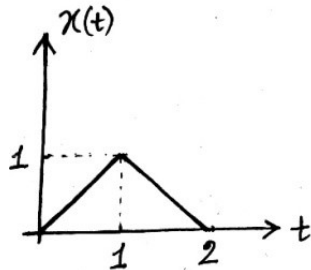


- (vi) The values of ω_n (natural frequency) and ξ (damping factor) for the second order system $G(s) = \frac{1}{0.25s^2 + s + 1}$ are
 (a) 2.5 and 1.0 (b) 1.0 and 0.5 (c) 2.0 and 1.5 (d) 2.0 and 1.0.
- (vii) For the causal discrete-time signal with Z transform $X(z) = \frac{z^2(2z-1.5)}{(z-1)(z-0.5)^2}$ the initial $x(0)$ and the final values $x(\infty)$ are
 (a) $x(0) = 2$ & $x(\infty) = -2$ (b) $x(0) = -2$ & $x(\infty) = 2$
 (c) $x(0) = 2$ & $x(\infty) = 2$ (d) $x(0) = -2$ & $x(\infty) = -2$
- (viii) For the current-force analogy in an electrical and mechanical (translation) systems, the reciprocal of inductance (in an electrical system) is analogous to the element
 (a) Viscous-friction coefficient (b) Velocity
 (c) Spring constant (d) Mass.
- (ix) The equivalent difference equation of the continuous time system $\dot{x}(t) = -0.25x(t) + 1$ with sampling time $T = \frac{\pi}{4}$ sec. (τ is the time constant of the above system) is
 (a) $x(k) = -0.25x(k-1) + 1$ (b) $x(k) = 0.75x(k-1) + 1$
 (c) $x(k) = -1.25x(k-1) + 2$ (d) $x(k) = -0.75x(k-1) + 1$.
- (x) The state space model of a 2nd-order system is described as $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ -3 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, the corresponding roots of the characteristic equation are
 (a) $-3 \pm 3j$ (b) $3 \pm 3j$ (c) $-3, 3$ (d) $-6, 0$.

Group - B

2. (a) Find whether a unit ramp signal $r(t)$ is an energy signal or a power signal.
 (b) Sketch the signal $f(t) = r(t+3) - r(t+2) - r(t-2) + r(t-3)$.
 (c) Find the convolution of the signals $x(t)$ and $h(t)$. Use graphical convolution method.



9. (a) Find the transfer function relating $V_a(s)$ to $\omega(s)$ of an electro-mechanical system as shown in Fig. Q 9(a) (assumptions: (i) armature reaction has been neglected (ii) frictional torque linearly related to velocity (iii) shaft torsional stiffness and motor inertia are neglected).

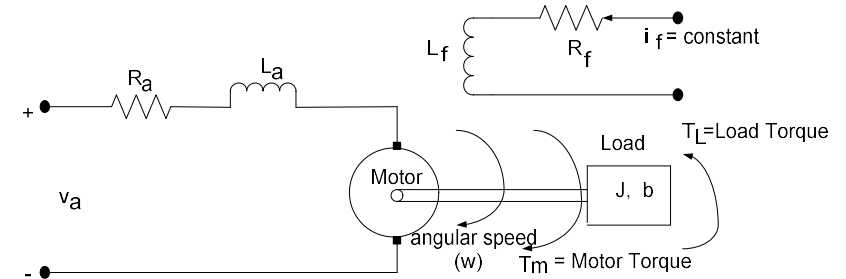


Fig.Q9(a)

- (b) Find the value of the state-vector $[x_1(t=3) \ x_2(t=3)]^T$ at time $t = 3$ second for state-space model with the following state transition ($\phi(t,0)$) matrices: $\phi(1,0) = \begin{bmatrix} 0.527 & 0.159 \\ -0.477 & -0.110 \end{bmatrix}$; $\phi(3,1) = \begin{bmatrix} 0.202 & 0.066 \\ -0.199 & -0.064 \end{bmatrix}$. Assume the input to the system is $u(t) = 0$ and initial value of the state vector $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 10 \\ -10 \end{bmatrix}$.

8 + 4 = 12

**SIGNALS & SYSTEMS
(ELEC 3103)**

Time Allotted : 3 hrs

Full Marks : 70

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.*

Candidates are required to give answer in their own words as far as practicable.

**Group – A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
 - (i) The normalized energy and the power of the signal $x(t) = e^{-\alpha} u(t)$ with $u(t) =$ unit step function and $\alpha = 0$ are,

(a) $E_x = \frac{1}{2}, P_x = 0$	(b) $E_x = 0, P_x = \frac{1}{2}$
(c) $E_x \rightarrow \infty, P_x = \frac{1}{2}$	(d) $E_x = \frac{1}{2}, P_x \rightarrow \infty$
 - (ii) The odd component of the signal $x(t) = u(t) - u(t-1)$ with $u(t) =$ unit step function is

(a) $x_o(t) = \begin{cases} 1 & \text{for } -1 \leq t \leq 2 \\ 0 & ; \text{ elsewhere} \end{cases}$	(b) $x_o(t) = \begin{cases} 0.5 & \text{for } -1 \leq t \leq 1 \\ 0 & ; \text{ elsewhere} \end{cases}$
(c) $x_o(t) = \begin{cases} 0.5 & \text{for } 0 \leq t \leq 1 \\ -0.5 & ; \text{ elsewhere} \end{cases}$	(d) $x_o(t) = \begin{cases} 0.5 & \text{for } 0 \leq t \leq 1 \\ -0.50 & ; -1 \leq t \leq 0 \end{cases}$
 - (iii) In Fourier series analysis of a periodic odd signal $x(t)$, there exist only “all odd-ordered harmonics with sine- components” when the given signal $x(t)$ having a characteristic as

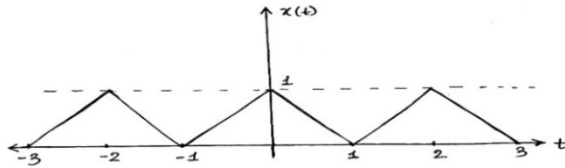
(a) Even-half periodic symmetry	(b) Even-periodic
(c) Odd-half periodic symmetry	(d) Odd- periodic.
 - (iv) The R.O.C of z – transform for the discrete signal $x(n) = 3^n u(n)$ (causal signal) is

(a) R.O.C : Complete z – complex plane	(b) R.O.C : $ z > 3$
(c) R.O.C : Outside the unit circle of z – plane	(d) R.O.C : $ z < 3$
 - (v) The impulse response of a causal linear continuous time system is denoted by $h(t)$, the system is bounded-input-bounded-output stable if the impulse response function $h(t)$ is

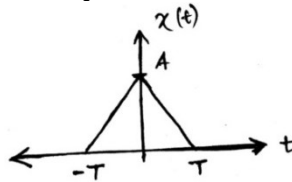
(a) absolutely square integrable	(b) absolutely integrable
(c) square integrable	(d) integrable.

- (d) Find out sequence of the signal $x(2n - 1)$ if $x(n) = \{3, 6, \overset{\downarrow}{1}, 0, 5\}$.
 $3 + 2 + 6 + 1 = 12$

3. (a) Find the trigonometric Fourier series of the signal shown below. And hence find out the exponential Fourier series coefficient.



- (b) Find the Fourier Transform of the following signal $x(t)$ and also sketch its amplitude and phase spectrum.



6 + 6 = 12

Group - C

4. (a) State and explain Shannon's sampling theorem with a suitable example. Obtain the discrete version of continuous time PID controller using backward difference approximations.
- (b) Find the z -transform of a non-causal signal $x(n) = 2^n u(n) + 3^n u(-n)$ and its R.O.C.

7 + 5 = 12

5. (a) (i) For the given z -transform pair $x(n) \leftrightarrow X(z)$ (for one-sided z -transform) and positive integer ' n ', show that z -transform of $(nx(n)) = -z \frac{dX(z)}{dz}$.

(ii) Determine the z -transform of $x(n) = n a^n u(n)$; where ' n ' is positive integer.

- (b) Determine the response of the discrete-time system (given below): $y(n) = x(n) + 0.5x(n-1) - 0.6y(n-1) - 0.08y(n-2)$ for a unit step input i.e. $x(n) = u(n)$ with zero initial conditions; using z -transform method.

(4 + 2) + 6 = 12

Group - D

6. (a) A single-input, single-output LTI system is input-output stable if, and only if, its transfer function has all its poles in the open left-half of the complex plane.
- (b) Find the impulse response of the system $H(s) = \frac{10}{(s+2)(s+4)}$ and hence determine whether the corresponding system is (i) causal (ii) stable based on the impulse response.

6 + 6 = 12

7. (a) A normalized (standard) second-order under-damped system $G(s) = \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$ is excited with a unit step input (assume zero initial conditions). Derive the expression for the output response and plot output response i.e. $y(t)$ -vs- time (t) . Show that the logarithm ratio of two successive overshoots (1st. and 2nd-overshoots) a and b can be expressed as $\frac{2\pi\xi}{\sqrt{1-\xi^2}}$.

- (b) The response of a normalized second-order system to a unit step was found to be oscillatory with zero initial conditions. The only measurements recorded were two successive overshoots (1st. and 2nd. overshoots), a and b equal to 0.4 and 0.08 respectively, and the period of oscillation $T_d = 0.4$ sec. Determine the values of the natural frequency (ω_n) and the damping ration (ξ) .

8 + 4 = 12

Group - E

8. (a) Define the following terms: State, State vector and State space of a dynamical system.
- (b) Convert the state-space model given below:
 $\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$ (state eq.); $y(t) = [1 \ 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ (output eq.) to an equivalent transfer function model $G(s)$ (assume initial conditions are zero) and hence compute natural frequency ω_n and damping ration ξ of the system. Is the given dynamic system is stable? - Justify your answer. Determine output response $y(t)$ of the system from transfer function model.

5 + 7 = 12