PHYSICS - II
(PHYS 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A generalized coordinate is called cyclic if
(a) the Lagrangian is explicitly independent of that coordinate
(b) the Hamiltonian is explicitly independent of that coordinate
(c) both (a) and (b)
(d) none of the above.

(ii) The degrees of freedom relation for a system of N particles with K constraints is given by
(a) N-K (b) 3N-K (c) N-3K (d) 3K-N.

(iii) The total wave function of a quantum particle
(a) is always complex
(b) is always real
(c) can be real or complex
(d) is a solution of the time independent Schrodinger equation.

(iv) Which one of the following functions is an eigen function of the operator $\frac{d^2}{dx^2}$?
(a) x (b) x^2 (c) e^{-x^2} (d) $cosx$.

(v) The average velocity of an electron at T = 0K in a metal in terms of velocity v_f at the Fermi level is
(a) $\frac{2}{3}v_f$ (b) $\frac{1}{3}v_f$ (c) $\frac{3}{2}v_f$ (d) $\frac{3}{4}v_f$.

(vi) At Fermi level, the probability of electron occupation at T > 0K is
(a) 0 (b) 1 (c) ½ (d) ¼.

(vii) The number of microstates for n particles in two compartments obeying MB statistics is
(a) n (b) 2^n (c) n^2 (d) 2^{n+1}.

(viii) The magnetic induction, $\vec{B}$, the magnetic field intensity $\vec{H}$, the magnetization $\vec{M}$, and the permeability of free space μ_0 are related as follows
(a) $\vec{B} = \mu_0(\vec{H} + \vec{M})$ (b) $\vec{B} = \mu_0\vec{H} + \vec{M}$
(c) $\vec{B} = \mu_0(\vec{H} - \vec{M})$ (d) $\vec{B} = \mu_0\vec{H} - \vec{M}$

(ix) In a p-type semiconductor, the donor level
(a) lies near the valence band edge
(b) lies near the conduction band edge
(c) lies halfway between the valence and conduction band edges
(d) does not exist.

(x) The number of possible arrangements of three fermions in three cells is
(a) 27 (b) 9 (c) 3 (d) 1.

Group – B

2. (a) What are generalized coordinates? Write down the Lagrangian and Lagrange's equation defining all the terms.

(b) Consider a particle sliding down a frictionless inclined plane. Write down Hamilton's equations for this system and solve them to find the equation of motion of the particle.

(c) The Lagrangian of a system is given by $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, where m and k are constants. What is the generalized momentum?

(d) Consider a simple pendulum with a mass m and string length l. Obtain Lagrange's equation of motion for this system.

(1 + 2) + (2 + 3) + 1 + 3 = 12

3. (a) What do you mean by normalization condition of a wave function? If a wave function of a quantum mechanical particle is given by

$\varphi(x,t) = A\,Sin\left(\frac{n\pi x}{L}\right)e^{-\frac{iEt}{\hbar}}$ for $0 \le x \le L$

$= 0$ otherwise

where, n = 0, 1, 2, 3, ... etc.

Find the value of A from normalization condition. Also find the value of x corresponding to maximum probability of finding the particle for the above system.

(b) Write down the 3-D Schrodinger equation for particle confined in a 3D potential box. Obtain the energy eigen function for it and show that the first excited state for the case has 3-fold degeneracy.

(c) Evaluate the commutator $[\hat{x}, \widehat{p_x}]$.

(1 + 2 + 2) + (1 + 3 + 1) + 2 = 12

Group – C

4. (a) Compare Maxwell-Boltzmann (MB), Fermi-Dirac (FD), and Bose-Einstein (BE) statistics mentioning at least three characteristics.

   (b) Three distinguishable particles are to be distributed into four energy levels having energies $0, E, 2E, 3E$ so that the total energy is $6E$. If the levels are nondegenerate, write down all the possible microstates and the corresponding macrostates. What is the thermodynamic probability of the most probable macrostate?

   (c) Using Fermi-Dirac statistics, calculate the concentration of electrons in the conduction band of an intrinsic semiconductor.

2 + (3 + 2 + 1) + 4 = 12

5. (a) State the fundamental postulate of statistical mechanics.

   (b) Write down the Fermi-Dirac distribution function and define Fermi level in metal at absolute zero and at finite temperature. Also show graphically the variation of f(E) with E for T = 0K and T > 0K in metal.

   (c) Develop the expression for the density of states available to the free electrons in a metal. Using this and the Fermi-Dirac distribution function, show that the average energy per electron at the absolute zero of temperature is given by $\frac{3}{5}E_F$ where E_F is the Fermi energy of the metal.

2 + (1 + 1 + 2) + (2 + 4) = 12

Group – D

6. (a) An assembly of parallel plate capacitors having area 6.45×10^{-4} m^2 and plate separation 0.002 m is connected with external potential 10 V. If the capacitor filled up with a dielectric material with dielectric constant value of 6.0, calculate
   (i) the value of capacitance.
   (ii) the amount of charge stored on each plate.
   (iii) the value of electric displacement vector.

   (b) Define electric polarizability? What are the different kinds of polarizabilities that may appear in a dielectric material?

   (c) Establish the relation concerning D, E and P. (Symbols have their usual meaning.)

   (d) The electric polarizability of an Argon atom is $1.43 \times 10^{-40}\ Farad - m^2$. Find the dielectric constant of Argon gas of density $1.8\ g/cm^3$.

(Atomic weight of Argon is $39.95\ gram/mole$, Avogadro number = 6.023×10^{23}.)

(1 + 2 + 1) + (1 + 1) + 3 + 3 = 12

7. (a) Briefly discuss the molecular origins of diamagnetism, paramagnetism, and ferromagnetism. Give an example of each.

   (b) Show that the induced magnetic moment of an atom in a diamagnetic material is opposite in direction to the applied external magnetic field.

   (c) Define H in terms of B and M for a linear magnetic material (symbols have their usual meaning).

(4.5 + 1.5) + 4 + 2 = 12

Group – E

8. (a) State and explain Bloch's theorem in one dimension.

   (b) Qualitatively explain the formation of energy bands in crystals.

   (c) An electron is moving in one dimension in a region where the potential is given by $V(x) = V_0\left[1 + cos\left({}^{4\pi x}/_{a}\right)\right]$, where V_0 and a are constants. Show that if $\psi(x)$ is an eigenfunction of the Hamiltonian then $\psi(x + a)$ is also an eigenfunction of the Hamiltonian with the same energy.

   (d) Write down the expression for the effective mass of an electron in a crystal. The energy – wave vector dispersion relation for a one-dimensional crystal of lattice constant a is given by $E(k) = E_0 - 2ak^2$ where E_0 is a constant. Find the expression of the effective mass of an electron in this crystal as a function of k.

(2 + 2) + 2 + 3 + (2 + 1) = 12

9. (a) What is the critical magnetic field for a superconductor? How does it vary with temperature?

   (b) Lead (Pb) gets transition to its superconducting state at 7.20 Kelvin. Lead has critical magnetic field at 0K is 65100 A/m, calculate its critical magnetic field at $-271^0\ C$.

   (c) Distinguish between type I and type II superconductors. Name some materials belonging to these two types of superconductors.

   (d) Establish London equation of superconductivity in terms of magnetic field induction and hence describe Meissner effect?

(2 + 1) + 2 + (2 + 1) + (1 + 3) = 12