(c) The language L_2 on the input alphabet $\sum = \{ 0, 1 \}$ is defined as follows:
$L_2 = \{ 0^p 1^m 0^q 1^m 0^r \mid p \geq 0, q \geq 0, r \geq 0, m \geq 0 \}$
Prove by using the Pumping Lemma for Regular Languages that L_2 is not regular.

(4 + 3) + 2 + 3 = 12

Group – D

6. (a) Consider the context-free grammar (CFG) G over {a, b}, with start symbol S, and with the following productions.
$S \rightarrow aaB \mid Abb$
$A \rightarrow a \mid aA$
$B \rightarrow b \mid bB$
Now answer the following:
i. What is L(G)? Give proper justifications to your answer
ii. Show that the given CFG is ambiguous.

(b) One of the following two languages is context-free, and the other is not. Identify which one is what. Justify your answer with proper reasons.
i. $L_4 = \{a^l b^m c^n \mid l, m, n >= 0, l + m >= n\}$
ii. $L_5 = \{a^l b^m c^n \mid l, m, n >= 0, l >= n \text{ and } m >= n\}$

(3 + 3) + (3 + 3) = 12

7. (a) Design a push down automata that will accept the following language:
$L = \{a^i b^i c^j d^j \mid i, j \geq 0\}$

(b) Find out the languages which are going to be accepted by a deterministic push down automata or non deterministic push down automata or both or none of the above.
i. $L(G) = \{ww^R \mid w \in (a, b)^*\}$, ii. $L(G) = \{a^n b^{n+2} \mid n > 0\}$
iii. $L(G) = \{a^n b^n c^m \mid n, m > 0\}$, iv. $L(G) = \{a^{n+m} b^{n+m} c^m \mid n, m > 0\}$
vi. $L(G) = \{a^n b^m \mid n, m > 0\}$.

7 + 5 = 12

Group – E

8. (a) Design a Turing machine for the following language: $\{0^n 1^m 0^n \mid m, n \geq 1\}$

(b) Write a short note on multi-tape and multi-head Turing machine.

7 + (2.5 + 2.5) = 12

9. (a) Briefly explain what is meant by a 'Universal Turing Machine'. Can every effective procedure be implemented by a Universal Turing Machine assuming the input is appropriately supplied?

(b) Design a Turing machine that decides the following language L_9 over the alphabet $\sum = \{0,1\}$:
$L_9 = \{w \mid w \text{ contains an equal number of 0's and 1's}\}$

5 + 7 = 12

FORMAL LANGUAGE & AUTOMATA THEORY
(CSEN 3101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If L_1 and L_2 are regular languages then which of the following is/are also regular language(s)?
(a) $L_1 \cup L_2$ (b) L_1L_2 (c) L_1^* (d) All of these.

(ii) Given that λ represents NULL, regular expression for the language $L = \{ w \in \{0, 1\}^* \mid w$ has no pair of consecutive zeros$\}$ is
(a) $(1 + 010)^*$ (b) $(01 + 10)^*$
(c) $(1 + 010)^* (0 + \lambda)$ (d) $(1 + 01)^* (0 + \lambda)$.

(iii) Recursively enumerable languages are not closed under:
(a) concatenation (b) complementation
(c) union (d) intersection.

(iv) For a Mealy machine if the input string is of length n then which of the following will be the length of the output string?
(a) n (b) n + 1 (c) n – 1 (d) None of these.

(v) Consider a CFG with the following productions:
$S \rightarrow aB \mid bA$, $A \rightarrow b \mid aS \mid bAA$, $B \rightarrow b \mid bS \mid aBB$
Which of the following type of strings of terminals does the above grammar generate?
(a) equal number of a's and b's
(b) odd number of a's and b's
(c) even number of a's and b's
(d) odd number of a's and even number of b's.

(vi) Given an arbitrary non-deterministic finite automaton (NFA) with n many states, then the maximum number of states in an equivalent deterministic finite automata (DFA) is:
(a) n! (b) 2^n (c) 2n (d) n^2.

(vii) Which of the following conversion is not possible (algorithmically)?
(a) regular grammar to context-free grammar
(b) nondeterministic FSA to deterministic FSA
(c) nondeterministic PDA to deterministic PDA
(d) nondeterministic TM to deterministic TM.

(viii) Which of the following statements is true?
(i) If a language is context free it can be always be accepted by a deterministic push-down automaton.
(ii) The union of two context free languages is context free.
(iii) The intersection of two context free languages is context free
(iv) The complement of a context free language is context free.
(a) (i)only (b) (ii)only (c) (iii) only (d) (iv) only.

(ix) Consider the following three languages:
$L_1 = \{a^n b^n c^m \mid m, n >= 1\}$
$L_2 = \{a^n b^m c^m \mid m, n >= 1\}$
$L_3 = \{a^n b^n c^n \mid n >= 1\}$
Which of the following statements is correct?
(a) L_1 and L_2 are CFL, but L_3 is not a CFL (b) $L_3 = L_1 \cap L_2$
(c) L_1 and L_2 are not CFL, but L_3 is a CFL (d) both (a) and (b).

(x) Which of the following problems is undecidable?
(a) Deciding if a given CFG is ambiguous
(b) Deciding if a given string is generated by a given CFG
(c) Deciding if a language generated by a given CFG is empty
(d) Deciding if a language generated by a given CFG is finite.

Group – B

2. (a) Consider the following deterministic finite automata (DFA) , where q1 is the starting state and q3, q5 are two final states)

δ	0	1
q_1	q_2	q_3
q_2	q_3	q_5
q_3	q_4	q_3
q_4	q_3	q_5
q_5	q_2	q_5

First determine the indistinguishable and distinguishable states for the given DFA and then construct a minimum deterministic finite automaton for the above mentioned DFA.

(b) State the differences between Mealy machine and Moore machine.

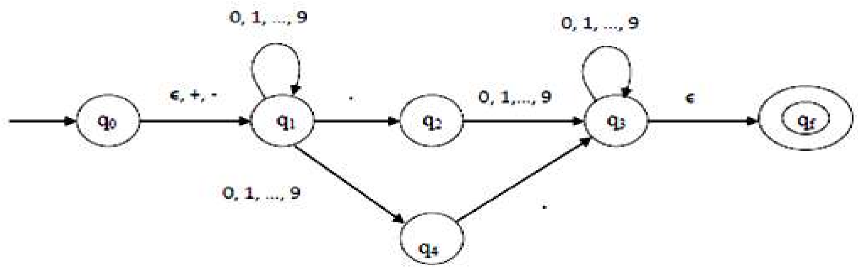
(c) Compute the null closure for each state:

States	λ (NULL)	a	b	c
p	-	p	q	r
q	p	q	r	-
r	q	r	-	p

(5 + 2) + 2 +3 = 12

3. (a) Explain the significance of non-determinism, where digital computers are completely deterministic in nature.

(b) Define ϵ-closure. Convert the following ϵ-NFA to its equivalent DFA using ϵ-closure

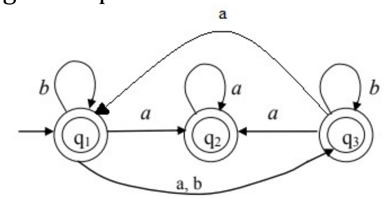


(c) Draw a Mealy machine on input alphabet $\sum = \{0, 1\}$ which can output EVEN or ODD as per the total number of 1's encountered is even or odd respectively.

2 + (2 + 6) + 2 = 12

Group – C

4. (a) Find out the regular expression for the following FSA:



(b) Check out whether you can generate regular grammar for the given language:
$L = \{0^m 1^n 2^p 3^q \mid m, n, p, q \geq 0 \text{ and } p \geq n\}$, Justify your answer.

7 + 5 = 12

5. (a) Let L_1 be the following language defined on the input alphabet $\sum = \{0, 1\}$.
$L_1 = \{\alpha \mid$ the string α in $\sum^*$ contains an even number of 0's and an odd number of 1's$\}$
i. Give the state transition diagram of a four-state deterministic finite automaton (DFA) *M* that recognizes L_1.
ii. Derive a regular (Type 3) grammar for L_1.

(b) Consider the set *S* of all strings α on the alphabet $\sum = \{0, 1, 2\}$ such that α contains at least one 0, at least two 1's and at least three 2's. Is *S* a regular set? Justify your answer.