B.TECH/CSE/3RD SEM/CSEN 2102/2018
DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A student can choose a computer project from one of the three lists. The three lists contain 23, 15 and 19 possible projects, respectively. No project is more than in one list. The number of possible projects to choose from is:
(a) 6555 (b) 57 (c) 36 (d) 76.

(ii) Which one of the following graph has a perfect matching?
(a) P_5 (b) C_7 (c) K_5 (d) C_8.

(iii) If $p \leftrightarrow q \equiv (p \rightarrow q) \wedge r$ then r is
(a) $p \rightarrow q$ (b) $\sim p$ (c) $q \rightarrow p$ (d) $\sim q$.

(iv) What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades A, B, C, D and F ?
(a) 26 (b) 29 (c) 30 (d) 31.

(v) How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?
(a) 1000000 (b) 970200 (c) 100 (d)970040.

(vi) Let G^* be the graph dual to the graph G, then the number of edges of G is equal to
(a) number of edges of G^*
(b) number of vertices of G^*
(c) number of regions determined by G^*
(d) number of components of G^*.

(vii) Which of the following is a non-planar graph?
(a) K_4 (b) $K_{2,2}$ (c) C_8 (d) $K_{4,4}$

(viii) The no. of integer solutions of $x_1 + x_2 + x_3 = 7$ where each $x_i \geq 0$, is
(a) $\frac{9!}{7!2!}$ (b) $\frac{9!}{7!}$ (c) $\frac{9!}{7!2!}$ (d) $\frac{9!}{7!2!}$.

(ix) $\sim(\sim p \vee \sim q) \equiv$
(a) $\sim p \wedge \sim q$ (b) $p \vee q$ (c) $\sim p \vee \sim q$ (d) $p \wedge q$.

(x) The vertex connectivity of exactly one of the following graphs is 1. Find it.
(a) $K_{4,4}$ (b) C_5 (c) P_8
(d) The Petersen graph.

Group – B

2. (a) Consider the following statements:
p :The flood destroys my house.
q:The fire destroys my house.
r: My insurance company will pay me.

Write down the following statements in p,q,r.
(i) "If the flood destroys my house or the fire destroys my house, then my insurance company will pay me".
(ii) "If my insurance company pays me, then the flood destroys my house or the fire destroys my house ".
(iii) "If neither the flood destroys my house nor the fire destroys my house then my insurance company will not pay me".
(iv) "If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house".

Find which one of (ii), (iii) and (iv) is
1. Converse
2. Inverse
3. Contrapositive
of the implication (i).

(b) Without using truth tables prove the following:
(i) $\sim\big(\sim(p \vee q) \vee (\sim p \wedge q)\big) \equiv p$.
(ii) $(p \wedge r) \vee (q \wedge r) \vee (\sim p \wedge (\sim q \wedge r)) \equiv r$

(4 + 2) + (3 + 3) = 12

3. (a) Find the disjunctive normal forms of the following statements.
(i) $\sim(\sim p \leftrightarrow q) \wedge r$ (ii) $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$.

(b) Check the validity of the following argument:
"If I pass B.Tech with high YGPA, I will be assured of a good job. If I am assured of a good job then my father will be happy. My father is not happy. Therefore I do not pass B.Tech with high YGPA."

(3 + 3) + 6 = 12

Group – C

4. (a) Determine all integer solutions to the equation: $x_1 + x_2 + x_3 + x_4 = 7$ where $x_i \geq 0, \forall 1 \leq i \leq 4$.

(b) Prove that any subset of size 6 from the set S={1,2,3,……….,9} must contain two elements whose sum is 10.

(c) Find the coefficient of $x^a y^b z^c w^d$ in the expansion of $(x+y+z+w)^n$. [Here a,b.c,d are integers ≥0, n=a+b+c+d],

5 + 3 + 4 = 12

5. (a) Find the solution to the recurrence relation: $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$.

(b) Prove the Principle of Inclusion-Exclusion for three sets A, B, C.

5 + 7 = 12

Group – D

6. (a) Prove Konig's Theorem: The chromatic number of a simple graph having at least one edge is 2 if and only if the graph contains no circuit of odd length.

(b) State the definition of the chromatic polynomial of a simple graph. Prove that the chromatic polynomial of any simple graph is a polynomial.

6 + 6 = 12

7. (a) Is the following polynomial the chromatic polynomial of a non-trivial simple graph having at least one edge? Justify your answer in detail.

(i) $x^5 - 5x^4 + 3x^3 - 4x^2 + 6x - 5$.

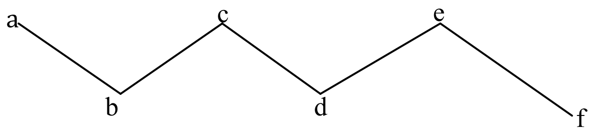
(ii) $x^5 - 5x^4 + 4x^3 - 6x^2 + 4x$.

(b) Let G be a simple connected planar graph with $n \geq 3$ vertices, e edges and f regions (faces). Prove that (i) $e \geq \frac{3}{2}f$ (ii) $e \leq 3n - 6$.

(3 + 3) +(4 +2) = 12

Group – E

8. (a) (i) State the definitions of a maximal matching, a maximum matching and a perfect matching. Find a maximal matching, a maximum matching and a perfect matching in the following graph.



(ii) How many perfect matchings can be found in K_4? Justify your answer.

(b) Let G be a simple connected planar graph having n vertices and e edges, drawn in a plane. Each region of G is bounded by k edges. Using the dual of G, prove that $k(n-2) = e(k-2)$.

(4 + 2) + 6 = 12

9. (a) Let G be a simple disconnected graph with the three components K_3, K_4 and P_4. Find the chromatic number of G. Justify your answer in detail.

(b) Find the number of perfect matching in $K_{8,8}$. Justify your answer and show your calculation in detail.

6 + 6 = 12