B.TECH/IT/5TH SEM/INFO 3133/2018
DISCRETE MATHEMATICS
(INFO 3133)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) $A \vee B$ is equivalent to which of the following?
(a) $\sim A \rightarrow \sim B$ (b) $\sim A \rightarrow B$ (c) $\sim B \rightarrow A$ (d) $\sim (A \rightarrow \sim B)$.

(ii) The remainder left when 3^{24} is divided by 7 is
(a) 1 (b) 2 (c) 4 (d) 6.

(iii) Minimal functionally complete set of connectives is
(a) $\{\sim, \vee, \wedge\}$ (b) $\{\sim, \vee\}$ (c) $\{\sim, \wedge, \rightarrow\}$ (d) $\{\sim, \vee, \leftrightarrow\}$.

(iv) The sum of co-efficients in the expansion of $(w + x + y + z)^5$ is
(a) 3^5 (b) 2^5 (c) 4^5 (d) 5^5.

(v) How many number of four digits can be formed with the digits 1,2,3,4 and 5 if repetition of digits is not allowed
(a) 30 (b)60 (c) 120 (d) 240.

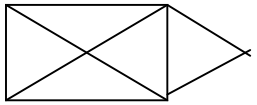
(vi) The generating function for the sequence $\{1,1,\frac{1}{2!},\frac{1}{3!},\frac{1}{4!},....\}$
(a) e^x (b) e^{-x} (c) $\log(1+x)$ (d) $(1-x)^{-1}$.

(vii) Which one of the following is not true for chromatic polynomial f (G,x) having chromatic number m
(a) $f(G,m) \neq 0$ (b) $f(G,m-1) \neq 0$
(c) $f(G,m+1) \neq 0$ (d) $f(G,m+2) \neq 0$.

(viii) Let p be a proposition 'He is intelligent' and q be a proposition 'He is tall'. Then $\sim p \wedge \sim q$ states that
(a) He is either intelligent or tall (b) He is neither tall nor intelligent
(c) He is not intelligent (d) He is intelligent and tall.

(ix) If a simple connected graph has at least one edge, then the sum of the coefficients in its chromatic polynomial is
(a) 1 (b) 2 (c) 3 (d) 0.

(x) Clique number of the graph



(a) 3 (b) 2 (c) 5 (d) 4.

Group – B

2. (a) Prove the following equivalence with the help of truth table.
(i) $\sim (p \vee q) \equiv \sim p \wedge \sim q$.
(ii) $(p \oplus q) \vee (p \downarrow q) \equiv p \uparrow q$.

(b) Without constructing the truth table prove the following (indicate the laws that are used).
$(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

(4 + 4) + 4 = 12

3. (a) Write quantified negated statement of each of the following statements:
(i) For $x \in R$, if $x^2 - 5x + 6 = 0$ then either $x = 3$ or $x = 5$.
(ii) For $a, b, c \in Z$, if $(a - b)$ is even and $(b - c)$ is even then $(a - c)$ is even.

(b) Write DNF of the following statement $p \rightarrow (p \wedge (q \rightarrow p))$.

3 + 3 + 6 = 12

Group – C

4. (a) Prove that, if d= gcd (x, y) then $\frac{x}{d}$ and $\frac{y}{d}$ are prime to each other.

(b) State and prove Fermat's Little Theorem.

6 + 6 = 12

5. (a) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$ then prove the following statement
(i) $a + c \equiv b + d (mod\ n)$
(ii) $ac \equiv bd (mod\ n)$

(b) Let S be a finite set. Prove that P(S), the power set of S, is a lattice with respect to the order relation $\subseteq$.

6 + 6 = 12

Group – D

6. (a) In how many ways can the integers 1 through 9 be permuted such that
(i) no odd integer will be in its natural position?
(ii) no even integer will be in its natural position?

(b) If m is an odd positive integer, prove that there exists a positive integer n such that m divides (2^n-1).

6 + 6 = 12

7. (a) Use the method of generating function to solve the recurrence relation
$a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$, $n \geq 0$,
given that $(\alpha_0 = 1, \alpha_1 = 4)$

(b) There are 3 piles of identical red, blue and green balls where each pile contains at least 10 balls. In how many ways can 10 balls be selected
(i) If there is no restriction?
(ii) If at least one red ball must be selected?
(iii) If at least one red ball, at least 2 blue balls and at least green balls must be selected?
(iv) If exactly one red ball must be selected?
(v) If exactly one red ball and at least one blue ball must be selected?
(vi) If at most one red ball is selected?

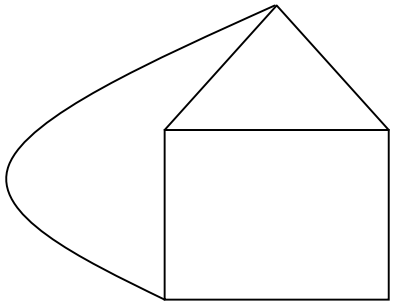
6 + 6 = 12

Group – E

8. (a) If G be a disconnected planar graph with n vertices, e edges, f faces and k components. Then prove that
$n - e + f = k + 1$.

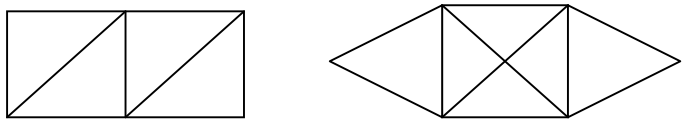
(b) A simple, connected, regular planar graph with 20 vertices are each of degree 3. Find the number of faces in the graph.

(c) Construct the dual graph of the following graph .



6 + 3 +3 = 12

9. (a) Find the clique number and independence number of the following two graphs



(b) Prove that the constant term in the chromatic polynomial is zero.

(c) The final examination of seven courses say with names a, b, c, d, e, f, g is to be scheduled in minimum possible days. Find the possible minimum number of days in which the examination can be scheduled if the following pair of courses have common students.

a: b, a: c, a: d, a: g
b: c, b: d, b: e, b: g
c: d, c: e, c: g
d: e, d: f
e: f, e: g
f: g

4 + 3 + 5 = 12